Week 1

Note Title 1/9/2006

Welcome !

Say:

X − # of years an American lives ← view it as a r.v.

Type of Question we ask:

X r.v. with pmf / pdf $f(x)$

we want to know, say,

$$\mu = E(X) = \begin{cases} \sum_i x_i f(x_i) & \text{dis.} \\ \int x f(x)\,dx & \text{cont.} \end{cases}$$

$$\sigma(X) = \begin{cases} \sqrt{\sum_i (x_i-\mu)^2 f(x_i)} \\ \sqrt{\int (x-\mu)^2 f(x)\,dx} \end{cases}$$

But we do not have $f(x)$.

(In practice, $f(x)$ is such a theoretical object anyway.

Nevertheless, people like measures such as $E(x)$, $V(x)$, ...)

Often, we have iid samples of the random variable.

$X_1, X_2, \cdots\cdots\cdots\cdots\cdots, X_n$

(i.i.d. stands for independent and identically distributed.)

It is like having a random # generator that allows us to generate many samples.

A <u>statistic</u> is any # whose value can be calculated from the sample data.

☆ VIEW IT AS A RANDOM VARIABLE.

Example.

$$\bar{X} = \frac{X_1 + X_2 + X_3}{3}$$

$$\tilde{X} = \text{median}(X_1, X_2, X_3)$$

$$\tilde{\tilde{X}} = \frac{\max(X_1, X_2, X_3) + \min(X_1, \cdots, X_n)}{2}$$

$$\bar{\sigma} = \sum_{i=1}^{n} (X_i - \bar{X})^2 / n$$

⋮

etc.

Ⓘ Proposition:

Let $X_1 \cdots X_n$ be a $\boxed{\text{random samples}}$ (iid) from a dist. with mean μ and standard deviation σ. Then

- $E(\bar{X}) = \mu_{\bar{X}} = \mu$

$$\bar{X} = \frac{X_1 + \cdots + X_n}{n} = \frac{1}{n}X_1 + \frac{1}{n}X_2 + \cdots + \frac{1}{n}X_n$$

- $V(\bar{X}) = \sigma_{\bar{X}}^2 = \sigma^2/n$ and $\sigma_{\bar{X}} = \sigma/\sqrt{n}$

- $E(X_1 + \cdots + X_n) = n\mu$

- $V(X_1 + \cdots + X_n) = n\sigma^2$

MeanEstimate.m illustrates this for the case

$$X_1, \cdots, X_n \overset{iid}{\sim} N(0, 1)$$

(V) Proposition

Let $X_1 \cdots X_n$ be a random sample from a $\boxed{\text{normal}}$ dist with mean μ and standard dev. σ.

Then $\bar{X}$ is normally distributed with mean μ and s.d. $\sigma/\sqrt{n}$.

(VI) CLT

Let $X_1 \cdots X_n$ be a r.v. from a dist. with mean μ and var. σ^2.

Then as $n \to \infty$

$\bar{X}$ has approximately a normal dist. with

$$\mu_{\bar{X}} = \mu$$

$$\sigma_{\bar{X}} = \sigma/\sqrt{n}$$

(VI) (Note: THE FOLLOWING INCLUDES (I) as a specifical case).

Let $X_1, \dots, X_n$ have mean values $\mu_1, \dots, \mu_n$ respectively. and variances $\sigma_1^2, \dots, \sigma_n^2$.

- whether or not $X_1, \dots, X_n$ are independent

$$E(a_1X_1 + \cdots + a_nX_n) = a_1\mu_1 + \cdots + a_n\mu_n$$

- If $X_1, \dots, X_n$ are independent

$$V(a_1X_1 + \cdots + a_nX_n) = a_1^2\sigma_1^2 + \cdots + a_n^2\sigma_n^2$$

$$\sigma_{a_1X_1 + \cdots + a_nX_n} = \sqrt{a_1^2\sigma_1^2 + \cdots + a_n\sigma_n^2}$$

- any $X_1, \dots, X_n$

$$V(a_1X_1 + \cdots + a_nX_n) = \sum_{i=1}^{n}\sum_{j=1}^{n} a_i a_j \operatorname{Cov}(X_i, X_j)$$