Chapter 2.5 Practice Problems

EXPECTED SKILLS:

- Know the derivatives of the 6 elementary trigonometric functions.
- Be able to use these derivatives in the context of word problems.

PRACTICE PROBLEMS:

1. Fill in the given table:

<table>
<thead>
<tr>
<th>$f(x)$</th>
<th>$f'(x)$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\sin x$</td>
<td></td>
</tr>
<tr>
<td>$\cos x$</td>
<td></td>
</tr>
<tr>
<td>$\tan x$</td>
<td></td>
</tr>
<tr>
<td>$\cot x$</td>
<td></td>
</tr>
<tr>
<td>$\sec x$</td>
<td></td>
</tr>
<tr>
<td>$\csc x$</td>
<td></td>
</tr>
</tbody>
</table>

2. Use the definition of the derivative to show that $\frac{d}{dx}(\cos x) = -\sin x$
   Hint: $\cos(\alpha + \beta) = \cos \alpha \cos \beta - \sin \alpha \sin \beta$

3. Use the quotient rule to show that $\frac{d}{dx}(\cot x) = -\csc^2 x$.

4. Use the quotient rule to show that $\frac{d}{dx}(\csc x) = -\csc x \cot x$.

5. Evaluate $\lim_{h \to 0} \frac{\tan \left( \frac{\pi}{3} + h \right) - \tan \left( \frac{\pi}{3} \right)}{h}$ by interpreting the limit as the derivative of a function at a particular point.

For problems 6-14, differentiate

6. $f(x) = 2\cos x + 4\sin x$

7. $f(x) = 5\cos x + \cot x$

8. $g(x) = 4\csc x + 2\sec x$

9. $f(x) = \sin x \cos x$

10. $f(x) = \frac{\sin^2 x}{\cos x}$
11. \( f(x) = x^3 \sin x \)
12. \( f(x) = \sec^2 x + \tan^2 x \)
13. \( f(x) = \frac{x + \sec x}{1 + \cos x} \)

For problems 14-17, compute \( \frac{d^2 y}{dx^2} \)

14. \( f(x) = \tan x \)
15. \( f(x) = \sin x \)
16. \( f(x) = \cos^2 x \)
17. \( f(x) = \sin^2 x + \cos^2 x \)

For problems 18-19, find all values of \( x \) in the interval \([0, 2\pi]\) where the graph of the given function has horizontal tangent lines.

18. \( f(x) = \sin x \cos x \)
19. \( g(x) = \csc x \)

20. Compute an equation of the line which is tangent to the graph of \( f(x) = \frac{\cos x}{x} \) at the point where \( x = \pi \).

21. Consider the graphs of \( f(x) = \sqrt{2} \cos x \) and \( g(x) = \sqrt{2} \sin x \) shown below on the interval \([0, \frac{\pi}{2}]\).

![Graphs of \( f(x) = \sqrt{2} \cos x \) and \( g(x) = \sqrt{2} \sin x \)](image)

Show that the graphs of \( f(x) \) and \( g(x) \) intersect at a right angle when \( x = \frac{\pi}{4} \). (Hint: Show that the tangent lines to \( f \) and \( g \) at \( x = \frac{\pi}{4} \) are perpendicular to each other.)
22. A 15 foot ladder leans against a vertical wall at an angle of $\theta$ with the horizontal, as shown in the figure below. The top of the ladder is $h$ feet above the ground. If the ladder is pushed towards the wall, find the rate at which $h$ changes with respect to $\theta$ at the instant when $\theta = 30^\circ$. Express your answer in $\text{feet/degree}$.

23. Use the Intermediate Value Theorem to show that there is at least one point in the interval $(0,1)$ where the graph of $f(x) = \sin x - \frac{1}{3}x^3$ will have a horizontal tangent line.

24. **Multiple Choice:** At how many points on the interval $[-\pi, \pi]$ is the tangent line to the graph of $y = 2x + \sin x$ parallel to the secant line which passes through the graph endpoints of the interval?

(a) 0  
(b) 1  
(c) 2  
(d) 3  
(e) None of these