Polar Coordinates: Tangent Lines, Arc Length, & Area

SUGGESTED REFERENCE MATERIAL:
As you work through the problems listed below, you should reference Chapter 10.3 of the recommended textbook (or the equivalent chapter in your alternative textbook/online resource) and your lecture notes.

EXPECTED SKILLS:
- Know how to compute the slope of the tangent line to a polar curve at a given point.
- Be able to find the arc length of a polar curve.
- Be able to calculate the area enclosed by a polar curve or curves.

PRACTICE PROBLEMS:
For problems 1-3, find the slope of the tangent line to the polar curve for the given value of $\theta$.

1. $r = \theta; \theta = \frac{\pi}{6}$
   \[
   \frac{\sqrt{3\pi + 6}}{6\sqrt{3} - \pi}
   \]
2. $r = 3 + 2\sin \theta; \theta = \frac{\pi}{6}$
   \[-5\sqrt{3}; \text{ Detailed Solution: [Here]}
3. $r = 1 - \sin 2\theta; \theta = \pi$
   \[-\frac{1}{2}
4. Consider the circle $r = 3\cos \theta$. Find all values of $\theta$ in $0 \leq \theta < \pi$ for which the curve has either a horizontal or vertical tangent line.
   \[
   \text{Vertical Tangent Lines when } \theta = 0 \text{ and } \theta = \frac{\pi}{2};
   \text{ Horizontal Tangent Lines when } \theta = \frac{\pi}{4} \text{ or } \theta = \frac{3\pi}{4}.
   
   For problems 5-7, find the arc length of the given curves

5. The entire circle $r = 4\sin \theta$.
   \[\frac{4\pi}{4}\]
6. The spiral \( r = e^{-\theta} \) for \( \theta \geq 0 \).

\[ \sqrt{2} \]

7. The entire cardioid \( r = 1 + \cos \theta \). (Hint: It may be useful to use symmetry and the identity \( \cos^2 \theta = \frac{1}{2}(1 + \cos(2\theta)) \))

8. Detailed Solution: [Here](#)

For problems 8-16, find the area of each of the specified regions.

8. The region in the 1st quadrant within the circle \( r = 3 \cos \theta \)

\[ \frac{9\pi}{8} \]

9. The region enclosed by the cardioid \( r = 3 + 3 \sin \theta \)

\[ \frac{27\pi}{2} \]

10. The region inside the circle \( r = 3 \) but outside the cardioid \( r = 1 + \cos \theta \)

\[ \frac{15\pi}{2} \]

11. The region inside the circle \( r = 3 \) but outside the cardioid \( r = 2 + 2 \cos \theta \)

\[ \frac{9\sqrt{3}}{2} + 2\pi \]

12. The region outside the circle \( r = 3 \) but inside the cardioid \( r = 2 + 2 \cos \theta \)

\[ \frac{9\sqrt{3}}{2} - \pi \]

13. The region in common between the two circles \( r = 3 \sin \theta \) and \( r = 3 \cos \theta \)

\[ \frac{9}{4} + \frac{9\pi}{8} \]

14. The region inside the circle \( r = 2 \) and to the right of the line \( r = \sec \theta \)

\[ \frac{4\pi}{3} - \sqrt{3} \]

15. The region enclosed by the rose \( r = 3 \cos 2\theta \)

\[ \frac{9\pi}{2} \]
16. The region enclosed by the rose \( r = 2 \sin 3\theta \)

17. Find the area of the shaded region (shown below) which is enclosed between the circle \( r = 2 \cos \theta \) and the cardioid \( r = 2 + 2 \cos \theta \).

\[ \frac{5\pi}{2} \]; Detailed Solution: [Here]

18. Consider the limaçon \( r = \sqrt{3} + 2\sqrt{3} \cos \theta \)

(a) Compute the area enclosed by the inner loop of the limaçon.

\( -\frac{9\sqrt{3}}{2} + 3\pi \); Detailed Solution: [Here]

(b) Compute the area enclosed between the outer and inner loops of the limaçon.

\( 9\sqrt{3} + 3\pi \); Detailed Solution: [Here]