Power Series

SUGGESTED REFERENCE MATERIAL:
As you work through the problems listed below, you should reference your lecture notes and the relevant chapters in a textbook/online resource.

EXPECTED SKILLS:

- Use sigma notation to write the Maclaurin series for a function \( f(x) \).
- Use sigma notation to write the Taylor series for a function \( f(x) \) about a specified \( x = x_0 \).
- Find the interval of convergence and the radius of convergence of a power series.
- Find the domain of a function that is expressed as a power series.

PRACTICE PROBLEMS:
For problems 1 & 2, use sigma notation to write the Maclaurin series for the given function.

1. \( f(x) = \ln(1 + x) \)
2. \( f(x) = x \cos x \)

For problems 3 & 4, use sigma notation to write the Taylor series for the given function about \( x = x_0 \).

3. \( f(x) = e^{2x}; \; x_0 = \ln 3 \)
4. \( f(x) = \sin x; \; x_0 = \frac{\pi}{2} \)

For problems 5 – 13, find the interval of convergence and the radius of convergence \( R \) for the power series.

5. \( x + x^2 + x^3 + x^4 + \ldots \)
6. \( x - \frac{x^2}{2} + \frac{x^3}{3} - \frac{x^4}{4} + \frac{x^5}{5} - \ldots \)
7. \( \sum_{k=0}^{\infty} \frac{x^k}{k!} \)
8. \( \sum_{k=0}^{\infty} (-1)^k \frac{x^{2k+1}}{2k + 1} \)
9. \( \sum_{k=0}^{\infty} \frac{(-5)^k x^k}{\sqrt{k + 10}} \)

10. \( \sum_{k=0}^{\infty} [(2k)! (2x + 1)^k] \)

11. \( \sum_{k=0}^{\infty} \left[ \left( \frac{2}{7} \right)^k (x + 4)^{k+1} \right] \)

12. \( \sum_{k=0}^{\infty} (-1)^k \frac{x^{2k+1}}{(2k + 1)!} \)

13. \( \sum_{k=2}^{\infty} \frac{(x - 3)^k}{k \ln k} \)

For problems 14 – 16, a function is represented as a power series. Find the domain of the function.

14. \( f(x) = \sum_{k=0}^{\infty} [(-1)^{k+1} (x - 2)^k] \)

15. \( f(x) = \sum_{k=0}^{\infty} (-1)^k \frac{x^{2k}}{(2k)!} \)

16. \( f(x) = \sum_{k=0}^{\infty} \frac{e^{(k^2)x^k}}{k!} \)