Differential Equations & First-Order Separable Equations

SUGGESTED REFERENCE MATERIAL:
As you work through the problems listed below, you should reference your lecture notes and the relevant chapters in a textbook/online resource.

EXPECTED SKILLS:
• Be able to verify that a given function is a solution to a differential equation.
• Be able to solve first-order separable equations by separating and integrating.
• Be able to solve initial-value problems for first-order separable equations.

PRACTICE PROBLEMS:
1. Verify that $y = x^2 + 1$ is a solution to the differential equation $y - \frac{dy}{dx} = (x - 1)^2$.

2. Find the value(s) of the constant $A$ for which $y = e^{Ax}$ is a solution to the differential equation $y'' + 5y' - 6y = 0$.

For problems 3-9, use separation of variables to solve the given differential equation. If possible, express your answer as an explicit function of $x$.

3. $\frac{dy}{dx} = \frac{x^2 - 1}{y^2}$

4. $\frac{dy}{dx} - \sqrt{xy} \ln x = 0$

5. $y' = yx^2$

6. $\frac{dy}{dx} - e^{-y} \sec^2 x = 0$

7. $\frac{dy}{dx} = xy^3$

8. $\frac{dy}{dx} = \frac{1}{(x^2 - 5x + 6)y}$

9. $\left(\frac{\sqrt{xy'}}{2 + y}\right) = 1$, for $x \neq 0$.

For problems 10-11, find the solution of the differential equation which satisfies the initial condition.
10. \( \frac{dy}{dx} = \frac{x^2 - 2}{y}, \ y(0) = 1 \)

11. \( \frac{dy}{dx} = \frac{\ln x}{xy^2}, \ y(e) = 1 \)

12. Find an equation of the curve that passes through the point \((0, 1)\) and whose slope at \((x, y)\) is \(xe^y\).