Monotone Sequences

SUGGESTED REFERENCE MATERIAL:
As you work through the problems listed below, you should reference your lecture notes and the relevant chapters in a textbook/online resource.

EXPECTED SKILLS:

• Understand what it means for a sequence to be increasing, decreasing, strictly increasing, strictly decreasing, eventually increasing, or eventually decreasing.

• Use an appropriate test for monotonicity to determine if a sequence is increasing or decreasing.

• Show that a sequence must converge to a limit by showing that it is monotone and appropriately bounded.

PRACTICE PROBLEMS:

1. Give an example of a convergent sequence that is not a monotone sequence.

2. Give an example of a sequence that is bounded from above and bounded from below but is not convergent.

For problems 3 and 4, determine if the sequence is increasing or decreasing by calculating \( a_{n+1} - a_n \).

3. \( \left\{ \frac{1}{4^n} \right\}_{n=1}^{+\infty} \)

4. \( \left\{ \frac{2n - 3}{3n - 2} \right\}_{n=1}^{+\infty} \)

For problems 5 and 6, determine if the sequence is increasing or decreasing by calculating \( \frac{a_{n+1}}{a_n} \).

5. \( \left\{ \frac{1}{4^n} \right\}_{n=1}^{+\infty} \)

6. \( \left\{ \frac{e^n - e^{-n}}{e^n + e^{-n}} \right\}_{n=1}^{+\infty} \)

For problems 7 and 8, determine if the sequence is increasing or decreasing by calculating the derivative \( a'_n \).
7. \( \left\{ \frac{1}{4^n} \right\}_{n=1}^{+\infty} \\
8. \left\{ \frac{\ln(2n)}{\ln(6n)} \right\}_{n=1}^{+\infty} \\

For problems 9 – 17, use an appropriate test for monotonicity to determine if the sequence increases, decreases, eventually increases, or eventually decreases.

9. \( \left\{ \frac{3n}{2n + 1} \right\}_{n=1}^{+\infty} \\
10. \left\{ n - \frac{1}{n} \right\}_{n=1}^{+\infty} \\
11. \left\{ \frac{n^2}{n!} \right\}_{n=1}^{+\infty} \\
12. \left\{ \frac{2n + 1}{(2n)!} \right\}_{n=1}^{+\infty} \\
13. \left\{ \frac{e^{\sqrt{n}}}{n} \right\}_{n=1}^{+\infty} \\
14. \left\{ e^n \pi^{-n} \right\}_{n=1}^{+\infty} \\
15. \left\{ \frac{3(n^2)}{(1000)^n} \right\}_{n=1}^{+\infty} \\
16. \left\{ \frac{n!}{n^n} \right\}_{n=1}^{+\infty} \\
17. \left\{ n^3 e^{-n} \right\}_{n=1}^{+\infty} \\

18. In the previous set of assigned problems it was shown that if the sequence

\[
\sqrt{30}, \sqrt{30 + \sqrt{30}}, \sqrt{30 + \sqrt{30 + \sqrt{30}}}, ...
\]

 converged to a limit, that limit was 6. Now we will show that the sequence is bounded above and increasing; thus, it must converge.

(a) Define the sequence recursively.
(b) Show that the sequence has an upper bound of 6.
(c) Show that the sequence is increasing by computing \( a_{n+1}^2 - a_n^2 \).