Consider the ODE \( \frac{dy}{dt} = k \left( 1 - \frac{y}{L} \right) y \) to be a function of \( y \), i.e. \( f(y) = k \left( 1 - \frac{y}{L} \right) y = k \left( y - \frac{y^2}{L} \right) \).

We want to maximize \( f(y) \), so using Calc I techniques:

\[ f'(y) = k \left( 1 - \frac{2y}{L} \right) = 0 \quad \Rightarrow \quad y = \frac{L}{2} \]

We can confirm this yields a maximum with the Second Derivative Test:

\[ f''(y) = \left( -\frac{2}{L} \right) < 0, \quad \text{so} \quad y = \frac{L}{2} \text{ maximizes } f(y) \]

and thus is when the population grows the fastest.