(a) Since the graph of \( y(x) \) goes through the origin and \( (\ln 2, 64) \) we know \( y(0) = 0 \) and \( y(\ln 2) = 64 \). So the BVP is

\[
\begin{align*}
25y &= 10y' - y'' \\
y(0) &= 0 \\
y(\ln 2) &= 64
\end{align*}
\]

(b) \( 25y = 10y' - y'' \) \( \iff \) \( y'' - 10y' + 25y = 0 \)

Auxiliary equation: \( m^2 - 10m + 25 = 0 \)

\( (m - 5)^2 = 0 \) \( \Rightarrow \) \( m = 5 \)

Solution to ODE:

\[ y = c_1 e^{5x} + c_2 x e^{5x} \quad [\text{This is } \#5] \]
\[ y(0) = 0 \implies 0 = c_1(1) + c_2(0) \]
\[ \implies c_1 = 0 \]

\[ y' (\ln 2) = 64 \implies 64 = 0 + c_2 (\ln 2) e^{5 \ln 2} \]
\[ e^{5 \ln 2} = (e^{\ln 2})^5 = 2^5 = 32 \]

So \[ 64 = 32 (\ln 2) c_2 \implies c_2 = \frac{2}{\ln 2} \]

Solution to BVP: \[ y = \frac{2}{\ln 2} x e^{5x} \]