Parametric Equations of Lines

SUGGESTED REFERENCE MATERIAL:

As you work through the problems listed below, you should reference Chapter 11.5 of the recommended textbook (or the equivalent chapter in your alternative textbook/online resource) and your lecture notes.

EXPECTED SKILLS:

- Be able to find the parametric equations of a line that satisfies certain conditions by finding a point on the line and a vector parallel to the line.
- Know how to determine whether two lines in space are parallel, skew, or intersecting. And, if the lines intersect, be able to determine the point of intersection.
- Know how to determine where a line intersects a surface.

PRACTICE PROBLEMS:

For problems 1-4, compute parametric equations of the line which satisfies the given conditions.

1. The line which passes through the point $(1, 0, -1)$ and is parallel to $\mathbf{v} = (1, -2, 0)$.

2. The line which passes through points $A(3, -6, 6)$ and $B(2, 0, 7)$.

3. The line which passes through the point $(-1, 2, 4)$ and is parallel to $L_1 = \begin{cases} x = 3 - 4t \\ y = 3 + 2t \\ z = t \end{cases}$

4. The line which passes through the point $(-2, 1, 4)$ and is parallel to both the $xy$-plane and the $xz$-plane.

5. Is the line which passes through points $A_1(1, 2, 3)$ and $B_1(5, 8, 9)$ parallel to the line which passes through points $A_2(-2, 5, 3)$ and $B_2(4, 14, 12)$?

6. Find the coordinates of the point at which the line $L_1 = \begin{cases} x = 3 - 6t \\ y = 3 + 3t \\ z = t \end{cases}$ intersects the given plane:

   (a) The $xy$-plane.
   (b) The $xz$-plane.
   (c) The $yz$-plane.
7. Find the coordinates of the points in 3-space where the line \( L_1 = \begin{cases} x = t \\ y = 1 + t \\ z = 1 - t \end{cases} \) intersects the sphere \( x^2 + y^2 + z^2 = 29 \).

For problems 8-11, determine whether the given lines intersect, are parallel, or are skew. If the lines intersect, find the point of intersection.

8. \( L_1 : x = 2 + 3t, y = 1 - 2t, z = 4 + 5t \)
\( L_2 : x = 3 - 6t, y = -2 + 4t, z = -1 - 10t \)

9. \( L_1 : x = 1, y = t, z = 2 - t \)
\( L_2 : x = 2 + 3t, y = 4 - 3t, z = t \)

10. \( L_1 : x = 1 - 2t, y = 14 + t, z = 5 - t \)
\( L_2 : x = t, y = 4 + 3t, z = 3 + t \)

11. \( L_1 : x = 2 + 5t, y = 4 - t, z = t + 1 \)
\( L_2 : x = 3 + 6t, y = 1 - t, z = t \)

12. Verify that the following lines are parallel. Then compute the distance between them.
(Hint: See HW 11.3 #10 or 11.4 #6.)
\( L_1 : x = 5 + 3t, y = 3 + 9t, z = 0 \)
\( L_2 : x = 1 + t, y = 3t, z = 1 \)

13. Two bugs are walking along lines in 3-space. At time \( t \), bug 1’s position is the point \((x, y, z)\) on the line \( L_1 = \begin{cases} x = 1 + 2t \\ y = 3 + 5t \\ z = 5 + 2t \end{cases} \) and bug 2’s position is the point \((x, y, z)\) on the line \( L_2 = \begin{cases} x = t \\ y = 11 - t \\ z = 4 + t \end{cases} \)

(a) Compute the distance between the bugs’ initial positions.

(b) At which point in space will the bugs’ paths intersect? (Note: the paths may not intersect at the same moment in time.)

14. Consider the point \( P(5, 3, 0) \) and the line \( L \) which contains points \( A(1, 0, 1) \) and \( B(2, 3, 1) \). This problem will show you another way to find the distance \( d \) between the point \( P \) and the line \( L \).

(a) Compute an equation of line \( L \).
(b) Compute a function $f(t)$ which gives the distance from the point $P$ to an arbitrary point on the line.

(c) The distance from the point $P$ to line $L$ is the shortest distance. Calculate the value of $t$ which minimizes the distance from the point $P$ to line $L$; that is, calculate the value of $t$ which minimizes $f(t)$ from part (b).

(d) Compute the distance from the point $P(5,3,0)$ to line $L$ by calculating the distance from this $P$ to the point on your the line which corresponds to your value of $t$ from part (c). Verify your answer with HW 11.3 #10(b).