Multivariable Functions

SUGGESTED REFERENCE MATERIAL:

As you work through the problems listed below, you should reference Chapter 13.1 of the recommended textbook (or the equivalent chapter in your alternative textbook/online resource) and your lecture notes.

EXPECTED SKILLS:

- Be able to describe and sketch the domain of a function of two or more variables.
- Know how to evaluate a function of two or more variables.
- Be able to compute and sketch level curves & surfaces.

PRACTICE PROBLEMS:

1. For each of the following functions, describe the domain in words. Whenever possible, draw a sketch of the domain as well.
   
   (a) \( f(x, y) = \sqrt{10 - x^2 - y^2} \)
   (b) \( f(x, y) = \arcsin (2x + y) \)
   (c) \( f(x, y, z) = \ln (36 - 4x^2 - 9y^2 - 36z^2) \)
   (d) \( f(x, y, z) = \sqrt{6 - 2x - 3y - z} \)

2. Let \( f(x, y) = 2xe^{3y} \). Compute the following.

   (a) \( f(4, 0) \)
   (b) \( f(1, \ln 2) \)

3. Suppose \( f(x, y) = \int_x^y (t^2 - 1) \, dt \). Compute the following.

   (a) \( f(-1, 2) \)
   (b) \( f(0, 2) \)

4. Suppose \( f(x_1, x_2, \ldots, x_n) = x_1 + 2x_2 + 3x_3 + \cdots + nx_n \). Determine \( f(1, 1, \ldots, 1) \).

5. Consider \( f(x, y) = x^2 + y^2 \). Compute \( f(x(t), y(t)) \) if \( x(t) = 1 + t \) and \( y(t) = 2 - 3t \)
6. Sketch the level curves $f(x, y) = k$, for the specified values of $k$.

   (a) $z = 2x - y; k = -2, -1, 0, 1, 2$
   (b) $z = y^2 - x^2; k = -2, -1, 0, 1, 2$

7. **Multiple Choice:** Which of the following graphs is the level curve of $f(x, y) = x^2 + 4y^2$ which passes through $P(-2, 0)$?

   (a) 
   (b) 
   (c) 
   (d) 
   (e)
8. Suppose \( f(x, y, z) = x^2 + y^2 - z^2 \). For each of the following, sketch the level surface \( f(x, y, z) = k \) corresponding to the indicated value of \( k \).

(a) \( k = 1 \)

(b) \( k = 0 \)

(c) \( k = -1 \)

9. Consider the contour map shown below.

(a) If a person were walking straight from point \( A \) to point \( B \), would s/he be walking uphill or downhill?

(b) Is the slope steeper at point \( B \) or point \( C \)?

(c) Starting at \( C \) and moving so that \( x \) remains constant and \( y \) decreases, will the elevation begin to increase or decrease?

(d) Starting at \( B \) and moving so that \( y \) remains constant and \( x \) increases, will the elevation begin to increase or decrease?
10. **Matching:** Each of the following contour plots were drawn on the window $[-3, 3] \times [-3, 3]$ in the $xy$-plane. Points with larger $z$-values are shaded in blue. Those with smaller $z$-values are shaded in red. Match each contour map (a-f) to an appropriate graph (I-VI).
11. **Multiple Choice:** Which of the following is a sketch of the domain of \( f(x, y) = \ln(xy - 1) + e^{x^2y} - y^8 \)?

- [ ] (a)  
- [ ] (b)  
- [ ] (c)  
- [ ] (d)  
- [ ] (e)  

12. Suppose that \( f(x, y) = x^2 + y^2 \). And, consider line \( L \) with parametric equations:  
\[
x(t) = \frac{\sqrt{2}}{2} t, \quad y(t) = \frac{\sqrt{2}}{2} t, \quad z(t) = 0.  
\]

Notice that this line in the \( xy \)-plane is parallel to the unit vector \( \vec{u} = \left( \frac{\sqrt{2}}{2}, \frac{\sqrt{2}}{2}, 0 \right) \).

(a) Now consider the image below. The red curve along \( f(x, y) \) is the curve that results from evaluating the function at points along \( L \). In other words, it is \( f(x(t), y(t)) \), where \( x(t) \) and \( y(t) \) are taken from the parametric equations of \( L \). Compute \( f(x(t), y(t)) \).
(b) Notice that your answer from part (a) is a single variable function of the variable $t$. Call it $g(t)$. Compute $\frac{d}{dt}(g(t))\bigg|_{t=1}$ and interpret your answer geometrically.