Local Linear Approximation

SUGGESTED REFERENCE MATERIAL:
As you work through the problems listed below, you should reference Chapter 13.4 of the recommended textbook (or the equivalent chapter in your alternative textbook/online resource) and your lecture notes.

EXPECTED SKILLS:
- Be able to compute the local linear approximation for a function of two or more variables at a given point.
- Be able to use a local linear approximation to estimate a given quantity.

PRACTICE PROBLEMS:

For problems 1-5, find the local linear approximation \( L(x, y) \) of the given function at the specified point.

1. \( f(x, y) = x^2 - y^2; \) \( P(1, 2) \)
2. \( f(x, y) = \frac{x + y}{x - y}; \) \( P(2, 1) \)
3. \( f(x, y) = e^x \sin y; \) \( P(\ln 3, \frac{\pi}{2}) \)
4. \( f(x, y) = \ln (x^2 - y^2); \) \( P(2, \sqrt{3}) \)
5. \( f(x, y) = \tan^{-1}\left(\frac{x}{y}\right); \) \( P(1, 1) \)

6. Find the local linear approximation of the function \( f(x, y) = \sqrt{32 - 3x^2 - y^2} \) at \( (1, 2) \) and use it to approximate \( f(0.98, 2.01) \).

7. Suppose that \( f(x, y) \) is a differentiable function at the point \( (2, 3) \) with \( f(2, 3) = 1, f_x(2, 3) = 5, \) and \( f_y(2, 3) = -2 \). Estimate \( f(1.98, 3.01) \).

8. Find the local linear approximation \( L(x, y, z) \) to \( f(x, y, z) = 3x^2 - 2y^2 + xz^3 \) at the point \( P(-1, 2, 1) \).

9. Verify that \( e^x \cos y \approx 1 + x \) for \( (x, y) \) near \( (0, 0) \).

10. Verify that \( (x + y)^3 \approx -16 + 12x + 12y \) for \( (x, y) \) near \( (1, 1) \).

11. At a particular point \( P(x_0, y_0) \), the local linear approximation of \( f(x, y) = xy + y^2 \) is \( L(x, y) = -15 + 3x + 8y \). What is the point \( P \)?