Local Linear Approximation

SUGGESTED REFERENCE MATERIAL:

As you work through the problems listed below, you should reference Chapter 13.4 of the recommended textbook (or the equivalent chapter in your alternative textbook/online resource) and your lecture notes.

EXPECTED SKILLS:

- Be able to compute the local linear approximation for a function of two or more variables at a given point.
- Be able to use a local linear approximation to estimate a given quantity.

PRACTICE PROBLEMS:

For problems 1-5, find the local linear approximation \( L(x, y) \) of the given function at the specified point.

1. \( f(x, y) = x^2 - y^2; \ P(1, 2) \)
   \[
   L(x, y) = 3 + 2x - 4y
   \]

2. \( f(x, y) = \frac{x + y}{x - y}; \ P(2, 1) \)
   \[
   L(x, y) = 3 - 2x + 4y
   \]

3. \( f(x, y) = e^x \sin y; \ P\left(\ln 3, \frac{\pi}{2}\right) \)
   \[
   L(x, y) = 3 - 3\ln 3 + 3x
   \]

4. \( f(x, y) = \ln (x^2 - y^2); \ P\left(2, \sqrt{3}\right) \)
   \[
   L(x, y) = -2 + 4x - 2\sqrt{3}y
   \]

5. \( f(x, y) = \tan^{-1}\left(\frac{x}{y}\right); \ P(1, 1) \)
   \[
   L(x, y) = \frac{\pi}{4} + \frac{1}{2}x - \frac{1}{2}y; \text{ Detailed Solution: Here }
   \]
6. Find the local linear approximation of the function \( f(x, y) = \sqrt{32 - 3x^2 - y^2} \) at \((1, 2)\) and use it to approximate \( f(0.98, 2.01) \).

\[
\begin{align*}
  f(x, y) &\approx L(x, y) = \frac{32}{5} - \frac{3}{5}x - \frac{2}{5}y \\
  \text{So, } f(0.98, 2.01) &\approx L(0.98, 2.01) = \frac{626}{125}
\end{align*}
\]

7. Suppose that \( f(x, y) \) is a differentiable function at the point \((2, 3)\) with \( f(2, 3) = 1 \), \( f_x(2, 3) = 5 \), and \( f_y(2, 3) = -2 \). Estimate \( f(1.98, 3.01) \).

\[
\begin{align*}
  f(x, y) &\approx L(x, y) = -3 + 5x - 2y \\
  \text{So, } f(1.98, 3.01) &\approx L(1.98, 3.01) = 0.88
\end{align*}
\]

8. Find the local linear approximation \( L(x, y, z) \) to \( f(x, y, z) = 3x^2 - 2y^2 + xz^3 \) at the point \( P(-1, 2, 1) \).

\[
L(x, y, z) = 8 - 5x - 8y - 3z
\]

9. Verify that \( e^x \cos y \approx 1 + x \) for \((x, y)\) near \((0, 0)\).

Show that the local linear approximation to \( f(x, y) = e^x \cos y \) at the point \((0, 0)\) is \( L(x, y) = 1 + x \).

10. Verify that \((x + y)^3 \approx -16 + 12x + 12y\) for \((x, y)\) near \((1, 1)\).

Show that the local linear approximation to \( f(x, y) = (x + y)^3 \) at the point \((1, 1)\) is \( L(x, y) = -16 + 12x + 12y \).

11. At a particular point \( P(x_0, y_0) \), the local linear approximation of \( f(x, y) = xy + y^2 \) is \( L(x, y) = -15 + 3x + 8y \). What is the point \( P \)?

\((2, 3)\); Detailed Solution: [Here]