Double Integrals Over Rectangular Regions

SUGGESTED REFERENCE MATERIAL:

As you work through the problems listed below, you should reference Chapter 14.1 of the recommended textbook (or the equivalent chapter in your alternative textbook/online resource) and your lecture notes.

EXPECTED SKILLS:

- Be able to compute double integral calculations over rectangular regions using partial integration.
- Know how to inspect an integral to decide if the order of integration is easier one way \((y \text{ first}, x \text{ second})\) or the other \((x \text{ first}, y \text{ second})\).
- Know how to use a double integral as the volume under a surface or find the area or a region in the \(xy\)-plane.

PRACTICE PROBLEMS:

For problems 1-4, evaluate the given iterated integral.

1. \[ \int_0^1 \int_0^2 (3x^3 - y^2 + 2) \, dx \, dy \]
   \[ \frac{46}{3} \]

2. \[ \int_0^2 \int_1^3 x^2y \, dy \, dx \]
   \[ \frac{32}{3} \]

3. \[ \int_0^{\ln 4} \int_0^{\ln 5} e^{x+y} \, dy \, dx \]
   \[ \frac{12}{3} \]

4. \[ \int_0^\pi \int_0^2 x \sin y \, dx \, dy \]
   \[ 3 \]
5. Consider \( f(x, y) = x^2 + y^2 \) and \( R : [0, 4] \times [0, 4] \).

(a) Estimate the volume bounded between the graph of \( f(x, y) \) and the \( xy \)-plane over the region \( R \) using 4 subrectangles of equal area and choosing the lower left hand corners as the sample points.

\[
64
\]

(b) Estimate the volume bounded between the graph of \( f(x, y) \) and the \( xy \)-plane over the region \( R \) using 4 subrectangles of equal area and choosing the upper right hand corners as the sample points.

\[
320
\]

(c) Estimate the volume bounded between the graph of \( f(x, y) \) and the \( xy \)-plane over the region \( R \) using 4 subrectangles of equal area and choosing the middle of the rectangle as the sample points.

\[
160
\]

(d) Compute the exact volume of the solid bounded between \( f(x, y) \) and the \( xy \)-plane over the region \( R \) using an appropriate double integral.

\[
\frac{512}{3}
\]

6. Each of the following iterated integrals represents the volume of a solid. Make a sketch of a solid whose volume is represented by the integral.

(a) \[
\int_{0}^{4} \int_{1}^{3} 5 \, dy \, dx
\]

This value of this integral can be thought of as the volume between the \( z = 5 \) plane and the \( xy \)-plane over the rectangle \([0, 4] \times [1, 3]\).
7. Use a double integral to find the volume of the solid which is bounded by the circular paraboloid $z = x^2 + y^2$ and the planes $z = 0, x = 0, x = 4, y = 0,$ and $y = 2.$ 

\[ \frac{160}{3} \]

8. Consider the rectangle $R$ in the $xy$-plane which has vertices $(0,1), (0,4), (3,1),$ and $(3,4).$

(a) Use a double integral to compute the area of $R.$

\[ A = \int_{0}^{3} \int_{1}^{4} 1 \, dy \, dx = \int_{1}^{4} \int_{0}^{3} 1 \, dx \, dy = 9 \]

(b) Verify your answer from part (a) by using an appropriate formula from geometry.

\[ A = bh = 3 \cdot 3 = 9 \]

9. By choosing a convenient order or integration, evaluate \( \int \int_{R} x \sec^2(xy) \sec^2 x \, dA \) where 

\[ R = \left\{ (x,y) : \frac{\pi}{4} \leq x \leq \frac{\pi}{3}, 0 \leq y \leq 1 \right\} \]

\[ \text{Detailed Solution: [Here]} \]