Double Integrals Over General Regions

SUGGESTED REFERENCE MATERIAL:

As you work through the problems listed below, you should reference Chapter 14.2 of the recommended textbook (or the equivalent chapter in your alternative textbook/online resource) and your lecture notes.

EXPECTED SKILLS:

• Be able to compute double integral calculations over rectangular regions using partial integration.

• Know how to inspect an integral to decide if the order of integration is easier one way (y first, x second) or the other (x first, y second).

• Know how to use a double integral to calculate the volume under a surface or find the area or a region in the xy-plane.

• Know how to reverse the order of integration to simplify the evaluation of a double integral.

PRACTICE PROBLEMS:

1. Consider the region $R$ shown below which is enclosed by $y = x^3$, $y = 0$ and $x = 1$.

[Diagram showing the region $R$]

Fill in the missing limits of integration.

(a) \[ \iint_R f(x, y) \, dA = \int_a^b \int_{g_1(x)}^{g_2(x)} f(x, y) \, dy \, dx \]

(b) \[ \iint_R f(x, y) \, dA = \int_{g_3(y)}^{g_4(y)} \int_{h_1(y)}^{h_2(y)} f(x, y) \, dx \, dy \]
2. Consider the region $R$ shown below which is enclosed by $y = \sqrt{4 - x^2}$ and $y = \frac{1}{2}(x+2)$.

(a) Set up $\int \int_{R} f(x, y) \, dA$ with the order of integration as $dy \, dx$

(b) Set up $\int \int_{R} f(x, y) \, dA$ with the order of integration as $dx \, dy$

For problems 3-7, evaluate the iterated integral. For some problems, it may be helpful to switch the order of integration.

3. $\int_{1}^{2} \int_{-x}^{x} \left( y^2 + 3xy + x^2 \right) \, dy \, dx$

4. $\int_{0}^{\pi/3} \int_{0}^{\sin x} y \cos x \, dy \, dx$

5. $\int_{0}^{1} \int_{0}^{x^3} \sqrt{1 - x^4} \, dy \, dx$

6. $\int_{0}^{1} \int_{0}^{1} \sqrt{1 - x^2} \, dx \, dy$

7. $\int_{0}^{\sqrt{\pi}/2} \int_{2y}^{\sqrt{\pi}} \sin \left( x^2 \right) \, dx \, dy$

8. Evaluate $\int \int_{R} (4x - 3y) \, dA$ where $R$ is the region enclosed by the circle $x^2 + y^2 = 1$.

9. Evaluate $\int \int_{R} xy^2 \, dA$ where $R$ is the triangular region enclosed by $y = 3x$, $y = \frac{x}{2}$, and $y = 1$. 

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10. Let $R$ be the region enclosed by $y = x^2$ and $y = 2x + 3$.

(a) Set up a double integral (or double integrals) with the order of integration as $dy \, dx$ which represents the area of $R$.

(b) Set up a double integral (or double integrals) with the order of integration as $dx \, dy$ which represents the area of $R$.

(c) Compute the area of $R$.

11. Use a double integral to find the volume of the solid in the first octant which is enclosed by the surface $3x + 6y + 2z = 12$ and the coordinate planes.

12. Consider the solid that enclosed by the cylinder $\frac{x^2}{9} + y^2 = 1$ and the planes $z = 0$ and $x + 2y + z = 4$. Use a double integral to compute the volume of this wedge.

13. Let $R$ be the region in the first quadrant of the $xy$ plane which is enclosed by $y = \sqrt{x}$, $x = 0$ and $y = 1$. Compute the volume of the solid which is bounded above by $z = xe^{x/y^2}$ and has $R$ as its base.