Math 121 - Exam 1 - 10/18/2013

NAME: Solutions

SECTION: _______________________

Directions:

• For the free response section, you must show all work. Answers without proper justification will not receive full credit. Partial credit will be awarded for significant progress towards the correct answer. Cross off any work that you do not want graded.

• You have 50 minutes to complete this exam. When time is called, STOP WRITING IMMEDIATELY.

• You may not use any electronic devices including (but not limited to) calculators, cell phones, or iPods.

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Free Response

Reminder: For each of the following problems, you must show all of your work to earn full credit. You may not use tools such as L’hopital’s rule or other applicable techniques which you may have learned elsewhere. You may only use techniques discussed in class up to and including chapter 2.4.

1. (11 Pts) Consider the quadratic function \( f(x) = ax^2 + bx + c \). Determine the values of the constants \( a \), \( b \), and \( c \) if \( f(1) = 9 \), \( f'(1) = 7 \), and \( f''(1) = 4 \).

\[
\begin{align*}
f(x) &= ax^2 + bx + c \quad & f'(x) &= 2ax + b \quad & f''(x) &= 2a \\
f''(1) &= 4 \quad & 2a &= 4 \quad & a &= 2 \\
f'(1) &= 7 \quad & 2(2)(1) + b &= 7 \quad & 4 + b &= 7 \quad & b &= 3 \\
f(1) &= 9 \quad & (2)(1)^2 + (3)(1) + c &= 7 \quad & 2 + 3 + c &= 9 \quad & c &= 4
\end{align*}
\]
2. (11 Pts) Use the limit definition of the derivative to compute $f'(x)$ if $f(x) = \sqrt{x}$
(No other method will be accepted, regardless of whether you obtain the correct derivative.)

$$f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$$

$$= \lim_{h \to 0} \frac{\sqrt{x+h} - \sqrt{x}}{h}$$

$$= \lim_{h \to 0} \frac{\sqrt{x+h} - \sqrt{x}}{h} \left( \frac{\sqrt{x+h} + \sqrt{x}}{\sqrt{x+h} + \sqrt{x}} \right)$$

$$= \lim_{h \to 0} \frac{(x+h) - (x)}{h(\sqrt{x+h} + \sqrt{x})}$$

$$= \lim_{h \to 0} \frac{1}{\sqrt{x+h} + \sqrt{x}}$$

$$= \frac{1}{2\sqrt{x}}$$

$$\therefore f'(x) = \frac{1}{2\sqrt{x}}$$
3. The graph of \( f(x) = \frac{x - 3}{2x - 1} \) is shown below.

(a) (6 Pts) Find the slope of the secant line which passes through the graph of \( f(x) \) at the points \( x = 1 \) and \( x = 3 \).

\[
\frac{f(1) - f(3)}{1 - 3} = \frac{-2}{-2} = 1 \quad \text{and} \quad \frac{f(3) - f(1)}{3 - 1} = \frac{0}{2} = 0
\]

\[
m = \frac{f(3) - f(1)}{3 - 1} = \frac{0 - (-2)}{2} = 1
\]
(b) (12 Pts) Find all value(s) of $x$ where the tangent line to the graph of $f(x) = \frac{x - 3}{2x - 1}$ is parallel to $y = 5x + 3$.

$$f'(x) = \frac{(2x-1)(1) - (x-3)(2)}{(2x-1)^2}$$

$$= \frac{2x-1 - 2x + 6}{(2x-1)^2}$$

$$= \frac{5}{(2x-1)^2}$$

$$\frac{5}{(2x-1)^2} = 5$$

$$\frac{5}{5} = (2x-1)^2$$

$$1 = (2x-1)^2$$

$$2x-1 = \pm 1$$

$$2x = 0 \quad \text{or} \quad 2x = 2$$

$$\boxed{x = 0} \quad \text{or} \quad \boxed{x = 1}$$
4. (10 Points) Evaluate \( \lim_{x \to +\infty} [\ln(2x + 3) - \ln(5x - 4)] \)

\[
\lim_{x \to +\infty} \left[ \ln(2x + 3) - \ln(5x - 4) \right] = \lim_{x \to +\infty} \ln \left( \frac{2x + 3}{5x - 4} \right)
\]

\[
= \ln \left( \lim_{x \to +\infty} \frac{2x + 3}{5x - 4} \right)
\]

\[
= \ln \left( \lim_{x \to +\infty} \frac{2x}{5x} \right)
\]

\[
= \ln \left( \lim_{x \to +\infty} \frac{2}{5} \right)
\]

\[
= \ln \left( \frac{2}{5} \right)
\]
Multiple Choice

Circle the letter of the best answer. Make sure your circles include just one letter. These problems will be marked as correct or incorrect; partial credit will not be awarded for problems in this section. All questions are worth 5 points.

5. Suppose \( f(x) = x^2 + \frac{1}{x^2} \). Which of the following is \( f'(1) \)?

- (a) 0
- (b) \( \frac{2}{3} \)
- (c) 1
- (d) \( \frac{3}{2} \)
- (e) 2

6. Evaluate \( \lim_{x \to 0} \frac{\sin^2(9x)}{x^2} \)

- (a) \( \frac{1}{9} \)
- (b) \( \frac{1}{81} \)
- (c) 1
- (d) 9
- (e) 81
7. Which of the following functions is continuous at $x = 0$ but not differentiable at $x = 0$?

(a) $f(x) = x^{-4/3}$

(b) $f(x) = x^{-1/3}$

(c) $f(x) = x^{2/3}$

(d) $f(x) = x^{4/3}$

(e) $f(x) = x^3$

8. On which of the following intervals does The Intermediate Value Theorem guarantee at least one solution to the equation $x^3 + x - 3 = 0$?

(a) $[0, 1]$

(b) $[0, 2]$

(c) $[2, 3]$

(d) $[-1, 0]$

(e) $[-2, -1]$
9. Compute \( \lim_{x \to 2^+} \frac{x^2 - 4}{x - 2} \)

(a) 0
(b) 1
(c) 4
(d) 32
(e) \( +\infty \)

10. Evaluate \( \lim_{x \to 0^-} \left( e^{1/x} \right) \)

(a) 0
(b) 1
(c) \( e \)
(d) \( +\infty \)
(e) \( -\infty \)

11. Suppose that \( F(x) \) is a differentiable function and define \( G(x) = (x^3 + 1)F(x) \). If \( F(1) = 4 \) and \( F'(1) = -3 \), what is the value of \( G'(1) \)?

(a) -9
(b) -1
(c) 6
(d) 8
(e) 12
12. Which of the following is an equation of the line which is tangent to the graph of 
\[ f(x) = 6x^3 - 4x + 1 \] at the point where \( x = 1 \)?

(a) \( y = 14x + 3 \)

(b) \( y = 14x - 11 \)

(c) \( y = 14x - 17 \)

(d) \( y = 18x - 15 \)

(e) \( y = 18x - 17 \)

13. Let 
\[ f(x) = \begin{cases} 
  x^2 + 5x + 1 & \text{if } x < -1 \\
  3 & \text{if } x = -1 \\
  2x - 1 & \text{if } x > -1 
\end{cases} \]

Why is \( f(x) \) not continuous at \( x = -1 \)?

(a) \( f(x) \) is not continuous at \( x = -1 \) because \( f(-1) \) does not exist.

(b) \( f(x) \) is not continuous at \( x = -1 \) because \( \lim_{x \to -1^-} f(x) \neq f(-1) \).

(c) \( f(x) \) is not continuous at \( x = -1 \) because \( \lim_{x \to -1^-} f(x) \) does not exist.

(d) \( f(x) \) is not continuous at \( x = -1 \) because \( \lim_{x \to -1^+} f(x) \) does not exist.

(e) \( f(x) \) is not continuous at \( x = -1 \) because \( \lim_{x \to -1^-} f(x) \) does not exist.
14. Recall the following definition:

**Definition:** A function $f(x)$ has a **vertical asymptote** of $x = a$ if at least one of the following is true:

- $\lim_{x \to a^-} f(x) = +\infty$
- $\lim_{x \to a^-} f(x) = -\infty$
- $\lim_{x \to a^+} f(x) = +\infty$
- $\lim_{x \to a^+} f(x) = -\infty$

The line $x = 1$ is a vertical asymptote for which of the following functions?

(I) $y = \frac{(x - 2)(x + 1)}{(x - 1)(x + 1)}$  
(II) $y = \frac{(x + 2)(x - 1)}{(x - 1)(x + 1)}$  
(III) $y = \frac{x}{\sin (\pi x)}$

(a) I only
(b) I and II only

(c) **I and III only**
(d) II and III only
(e) I, II, and III