Math 121 - Exam 1 - 10/20/2014

Name: ___________________________  Section: ______________

<table>
<thead>
<tr>
<th>Section</th>
<th>Class Times</th>
<th>Day</th>
<th>Instructor</th>
<th>Section</th>
<th>Class Times</th>
<th>Day</th>
<th>Instructor</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>09:00 AM - 09:50 AM</td>
<td>M T W F</td>
<td>Li, Hulan</td>
<td>15</td>
<td>11:00 AM - 11:50 AM</td>
<td>M T W F</td>
<td>Perlstadt, Marcia</td>
</tr>
<tr>
<td>2</td>
<td>09:00 AM - 09:50 AM</td>
<td>M T W F</td>
<td>Lee, Hwan Yong</td>
<td>16</td>
<td>04:00 PM - 04:50 PM</td>
<td>M T W F</td>
<td>Aran, Jason</td>
</tr>
<tr>
<td>3</td>
<td>09:00 AM - 09:50 AM</td>
<td>M T W F</td>
<td>Papadopoulos, Dimitrios</td>
<td>21</td>
<td>12:00 PM - 01:50 PM</td>
<td>M W</td>
<td>White, Richard</td>
</tr>
<tr>
<td>5</td>
<td>02:00 PM - 02:50 PM</td>
<td>M T W F</td>
<td>Swartz, Kenneth</td>
<td>22</td>
<td>02:00 PM - 02:50 PM</td>
<td>M T W F</td>
<td>Grinshpan, Anatolii</td>
</tr>
<tr>
<td>6</td>
<td>09:00 AM - 09:50 AM</td>
<td>M T W F</td>
<td>Grinshpan, Anatolii</td>
<td>23</td>
<td>01:00 PM - 01:50 PM</td>
<td>M T W F</td>
<td>Papadopoulos, Dimitrios</td>
</tr>
<tr>
<td>7</td>
<td>10:00 AM - 10:50 AM</td>
<td>M T W F</td>
<td>Li, Hulan</td>
<td>24</td>
<td>01:00 PM - 01:50 PM</td>
<td>M T W F</td>
<td>Perline, Ronald</td>
</tr>
<tr>
<td>8</td>
<td>10:00 AM - 10:50 AM</td>
<td>M T W F</td>
<td>Perlstadt, Marcia</td>
<td>25</td>
<td>05:00 PM - 05:50 PM</td>
<td>M T W F</td>
<td>Aran, Jason</td>
</tr>
<tr>
<td>9</td>
<td>12:00 PM - 12:50 PM</td>
<td>M T W F</td>
<td>Papadopoulos, Dimitrios</td>
<td>26</td>
<td>11:00 AM - 11:50 AM</td>
<td>M T W F</td>
<td>Lee, Hwan Yong</td>
</tr>
<tr>
<td>11</td>
<td>01:00 PM - 01:50 PM</td>
<td>M T W F</td>
<td>Aran, Jason</td>
<td>27</td>
<td>12:00 PM - 12:50 PM</td>
<td>M T W F</td>
<td>Zhang, Aljun</td>
</tr>
</tbody>
</table>

The following rules apply:

- **This is a closed-book exam.** You may not use any books or notes on this exam.

- **For free response questions, you must show all work.** Answers without proper justification will not receive full credit. Partial credit will be awarded for significant progress towards the correct answer. Cross off any work that you do not want graded.

- **For multiple choice questions, circle the letter of the best answer.** Make sure your circles include just one letter. These problems will be marked as correct or incorrect; partial credit will not be awarded for problems in this section.

- **You have 50 minutes to complete this exam.** When time is called, stop writing immediately and turn in your exam to the nearest proctor.

- **You may not use any electronic devices including (but not limited to) calculators, cell phone, or iPods.** Using such a device will be considered a violation of the university’s academic integrity policy and, at the very least, will result in a grade of 0 for this exam.

- **You may only use techniques discussed in class up to and including chapter 2.4.** You should not use l'Hôpital’s Rule or other applicable techniques which you may have learned elsewhere.

<table>
<thead>
<tr>
<th>Page</th>
<th>Points</th>
<th>Score</th>
</tr>
</thead>
<tbody>
<tr>
<td>2</td>
<td>15</td>
<td></td>
</tr>
<tr>
<td>3</td>
<td>15</td>
<td></td>
</tr>
<tr>
<td>4</td>
<td>15</td>
<td></td>
</tr>
<tr>
<td>5</td>
<td>15</td>
<td></td>
</tr>
<tr>
<td>6</td>
<td>10</td>
<td></td>
</tr>
<tr>
<td>7</td>
<td>10</td>
<td></td>
</tr>
<tr>
<td>8</td>
<td>10</td>
<td></td>
</tr>
<tr>
<td>9</td>
<td>10</td>
<td></td>
</tr>
</tbody>
</table>

Total: 100
Part I: Free Response

1. (15 points) For what value(s) of $x$ is the tangent line to the graph of $f(x) = \frac{4x + 3}{x - 2}$ parallel to $y = -11x + 9$?

$$f'(x) = \frac{(x-2)(4) - (-1)(4x+3)}{(x-2)^2}$$

$$= \frac{4x - 8 - 4x - 3}{(x-2)^2}$$

$$= \frac{-11}{(x-2)^2}$$

$$f'(x) = -11$$

$$\frac{-11}{(x-2)^2} = -11$$

$$(x-2)^2 = 1$$

$$x - 2 = \pm 1$$

$$x = 3, 1$$
2. (15 points) Suppose \( f(x) = \frac{5}{x} \). Use the definition of the derivative to compute \( f'(x) \).

(No other method will be accepted.)

\[
f'(x) = \lim_{h \to 0} \frac{\frac{5}{x+h} - \frac{5}{x}}{h}
\]

\[
= \lim_{h \to 0} \frac{1}{h} \left( \frac{5x - 5(x+h)}{x(x+h)} \right)
\]

\[
= \lim_{h \to 0} \frac{1}{h} \left( \frac{-5h}{x(x+h)} \right)
\]

\[
= \lim_{h \to 0} \frac{-5}{x(x+h)}
\]

\[
= -\frac{5}{x(x)}
\]

\[
f'(x) = -\frac{5}{x^2}
\]
3. Evaluate the following limits. If a limit does not exist, write “+∞,” “−∞,” or “DNE” (whichever is most appropriate).

(a) (8 points) \( \lim_{x \to -\infty} \frac{e^x}{e^x - 7} \)

Let \( t = e^x \).

As \( x \to -\infty \), \( t \to 0 \)

\[
\lim_{x \to -\infty} \frac{e^x}{e^x - 7} = \lim_{t \to 0} \frac{t}{t - 7} = \frac{0}{-7} = 0
\]

(b) (7 points) \( \lim_{x \to \frac{\pi}{2}} \ln(\sec^2 x) \)

\[
\lim_{x \to \frac{\pi}{2}} \ln(\sec^2 x) = \ln\left[ \lim_{x \to \frac{\pi}{2}} \sec^2 x \right] = \ln\left[ \left( \frac{\sqrt{2}}{1} \right)^2 \right] = \ln 2
\]
4. (15 points) Consider the following function:

\[ f(x) = \begin{cases} 
5\sin(ax) / x & \text{if } x > 0 \\
4a^2 + 4x & \text{if } x \leq 0 
\end{cases} \]

Find all non-zero value(s) of \( a \) so that \( f(x) \) is continuous at \( x = 0 \).

\[
\lim_{x \to 0^-} f(x) = \lim_{x \to 0^-} \frac{5\sin(ax)}{x} = \frac{5\sin(a \cdot 0)}{0} = 0
\]

\[
\lim_{x \to 0^+} f(x) = \lim_{x \to 0^+} (4a^2 + 4x) = 4a^2
\]

\[
f(0) = 4a^2 + 0 = 4a^2
\]

\[
\lim_{x \to 0^-} f(x) = \lim_{x \to 0^+} f(x) = f(0)
\]

\[
5a = 4a^2
\]

\[
4a^2 - 5a = 0
\]

\[
a(4a - 5) = 0
\]

\[
a = 0, \frac{5}{4}
\]

\[
a = \frac{5}{4}
\]
Part II: Multiple Choice

5. (5 points) Which of the following is the slope of the tangent line to the graph of \( f(x) = x^2 - 4\sqrt{x} \) at \( x = 4 \)?

(a) 6

(b) 7

(c) 8

(d) 9

(e) 12

6. (5 points) Evaluate \( \lim_{x \to +\infty} \ln \left( \frac{1}{x^2} \right) \).

(a) 0

(b) 1

(c) \(e\)

(d) \(-\infty\)

(e) \(+\infty\)
7. (5 points) Suppose \( f(x) = \frac{x^2 + x - 20}{x - 4} \) for \( x \neq 4 \). What value should be assigned to \( f(4) \) to make \( f(x) \) continuous at \( x = 4 \)?

(a) \(-1\)
(b) \(0\)
(c) \(5\)
(d) \(9\)
(e) \(20\)

8. (5 points) An equation of the line tangent to the curve \( y = 3x^2 - 7x \) at the point where \( x = 1 \) is:

(a) \(y = -x\)
(b) \(y = -x + 1\)
(c) \(y = -x - 3\)
(d) \(y = 6x - 7\)
(e) \(y = 6x - 4\)
9. (5 points) Consider the graph of \( y = f(x) \) shown below.

At which of the following value(s) of \( x \) does \( f'(x) \) not exist?

(a) \( x = -2 \) and \( x = 0 \)

(b) \( x = -2 \) and \( x = 1 \)

(c) \( x = -4 \) and \( x = 0 \)

(d) \( x = -4 \) and \( x = -2 \)

(e) \( x = -4, x = -2, \) and \( x = 1 \)

10. (5 points) Suppose \( g(x) \) is a differentiable function with \( g(2) = 5 \) and \( g'(2) = -3 \). If \( y = x^3 g(x) \), what is \( \frac{dy}{dx} \bigg|_{x=2} \) ?

(a) 36

(b) 20

(c) 10

(d) -12

(e) -36
11. (5 points) Find the average rate of change of \( f(x) = x^2 + 6 \) from \( x = -3 \) to \( x = 1 \).

(a) 2  
(b) 0  
(c) \( -2 \)  
(d) -4  
(e) -6

12. (5 points) The vertical and horizontal asymptotes of \( f(x) = \frac{x^2 + 2x - 3}{x^2 - 1} \) are:

(a) \( x = -1, y = 1 \)  
(b) \( x = -1, y = 0 \)  
(c) \( x = -1, x = 1, y = 1 \)  
(d) \( x = -1, x = 1, y = 0 \)  
(e) \( x = -1, x = 1, y = -3, y = 1 \)