Sample Questions

The following questions are not comprehensive of the material that you are responsible for knowing. These problems are meant to offer you some practice with multiple choice questions. You should refer back through the assigned homework exercises on the main course webpage for a full list of "Expected Skills" as well as a more comprehensive study guide.

Multiple Choice: Circle the letter of the best answer.

1. What is the natural domain of the function \( f(x) = \frac{(x + 2)^2 \ln(x - 1)}{\sqrt{16 - x^2}} \)?
   (a) (1, 4)
   (b) [1, 4)
   (c) (1, 4]
   (d) (−4, 4)
   (e) [−4, 4]

2. What is \( \lim_{x \to -\infty} \frac{2x - 7}{\sqrt{x^2 + x - 1}} \)?
   (a) \( -\infty \)
   (b) \(-2\)
   (c) \(-1\)
   (d) \(0\)
   (e) \(2\)

3. What is \( \lim_{x \to 1^+} \frac{x}{\ln(x)} \)?
   (a) \(0\)
   (b) \(1\)
   (c) \(e\)
   (d) \(e^{-1}\)
   (e) \(+\infty\)
4. \[
f(x) = \begin{cases} 
  x^2, & \text{if } x < -2 \\
  4, & -2 < x \leq 1 \\
  6 - x, & x > 1 
\end{cases}
\]

For the above function \( f \), which of the following statements is true?

(a) \( f \) is continuous everywhere.
(b) If \( f(-2) \) were defined to be 4, then \( f \) would be continuous everywhere.
(c) \( f \) is discontinuous at only \( x = -2 \).
(d) \( f \) is discontinuous at only \( x = 1 \).
(e) \( f \) is discontinuous at \( x = -2 \) and \( x = 1 \).

5. If \( f(x) = e^{x^2+2x} \), then \( f'(0) = \)
   
   (a) 1
   (b) 2
   (c) 3
   (d) 4
   (e) 6

6. What is the equation of the tangent line to \( f(x) = \tan^{-1}(2x) \) at \( x = 0 \)?
   
   (a) \( y = x \)
   (b) \( y = x + 1 \)
   (c) \( y = x - 1 \)
   (d) \( y = 2x \)
   (e) \( y = 2x - 1 \)

7. If \( y = \tan^2(x) \), what is \( \frac{dy}{dx} \)?
   
   (a) \( \sec^2(x) - 1 \)
   (b) \( \sec^2(x) \)
   (c) \( 2\tan(x)\sec(x) \)
   (d) \( 2\tan(x)\sec^2(x) \)
   (e) \( 2\tan^2(x)\sec^2(x) \)
8. Which of the following is the best local linear approximation for \( f(x) = \tan(x) \) near \( x = \frac{\pi}{4} \)?

(a) \( 1 + \left( x - \frac{\pi}{4} \right) \)
(b) \( 1 + \frac{1}{2} \left( x - \frac{\pi}{4} \right) \)
(c) \( 1 + \sqrt{2} \left( x - \frac{\pi}{4} \right) \)
(d) \( 1 + 2 \left( x - \frac{\pi}{4} \right) \)
(e) \( 2 + 2 \left( x - \frac{\pi}{4} \right) \)

9. If \( x^2 - y^2 = 10 \), what is \( \frac{d^2 y}{dx^2} \)?

(a) \( \frac{y^2 - x^2}{y^3} \)
(b) \( \frac{y - x}{y^3} \)
(c) \( \frac{y - x}{y^2} \)
(d) \( \frac{x}{y} \)
(e) \( -\frac{10}{y^2} \)

10. What is \( \lim_{x \to 0} \frac{e^x - 1}{\tan(x)} \)?

(a) \( -1 \)
(b) \( 0 \)
(c) \( 1 \)
(d) \( 2 \)
(e) The limit does not exist.
11. Which of the following is the derivative of \( f(x) = \ln\left(\frac{x + 1}{x}\right) \)?

(a) \(-\frac{(x + 1)}{x^3}\)
(b) \(-\frac{x}{x + 1}\)
(c) \(-\frac{x}{x + 1}\)
(d) \(-\frac{1}{x(x + 1)}\)
(e) \(-\frac{1}{x^2(x + 1)}\)

12. If \( \lim_{x \to +\infty} f(x) = \lim_{x \to +\infty} g(x) = +\infty \) and \( f'(x) = 1 \) and \( g'(x) = e^x \), what is \( \lim_{x \to +\infty} \frac{f(x)}{g(x)} \)?

(a) \(-1\)
(b) \(0\)
(c) \(1\)
(d) \(e\)
(e) The limit does not exist.

13. For the function \( f(x) = \sin^2(x) + \cos(x) \) on the interval \([-\pi, 0]\), which of the following is the \( x \)-coordinate of the absolute maximum?

(a) \(-\pi\)
(b) \(-\frac{\pi}{3}\)
(c) \(-\frac{\pi}{6}\)
(d) \(-\frac{\pi}{2}\)
(e) \(\pi\)
14. The rational function \( f(x) = \frac{x^2}{x^2 + x - 12} \) has the asymptotes

(a) \( x = -4, x = 3 \) only
(b) \( x = -4, x = 3, y = 1 \)
(c) \( x = 4, x = -3, y = 1 \)
(d) \( x = -4, x = 3 \) only
(e) \( x = -4, x = 3, x = 1 \)

15. If the function \( f \) is not differentiable at \( x = 0 \), then which of the following MUST be true?

(a) \( f(0) \) is undefined.
(b) \( f \) is NOT continuous at \( x = 0 \).
(c) There is a horizontal tangent line to the graph of \( y = f(x) \) at \( x = 0 \).
(d) There is a vertical tangent line to the graph of \( y = f(x) \) at \( x = 0 \).
(e) \( \lim_{h \to 0} \frac{f(0 + h) - f(0)}{h} \) does not exist.

16. The function \( y = x^3 - 6x^2 + 9x - 4 \) has a relative maximum at the point \( (a, b) \) and a relative minimum at the point \( (c, d) \). What is the value of \( a + b + c + d \)?

(a) \(-4\)
(b) \(-1\)
(c) \(0\)
(d) \(1\)
(e) \(4\)

17. The curve \( y = -3x^2 \ln(x) \) has a single inflection point at \( x = e^k \). What is the value of \( k \)?

(a) \( \frac{3}{2} \)
(b) \( \frac{1}{2} \)
(c) \( \frac{1}{2} \)
(d) \( \frac{3}{2} \)
(e) \(2\)
18. The graph above shows the graph of $f'(x)$, the derivative of $f$. For what $x$-values would the graph of $f(x)$ be concave up?

(a) $x < -3$
(b) $x < -2$ and $x > 0$
(c) $-2 < x < 0$
(d) $-3 < x < 0$
(e) $x > 0$

Free Response: You must show all work and use correct notation to earn full credit.

19. Using the definition of the derivative, calculate $f'(x)$ for $f(x) = \frac{2}{3 - x}$.

20. In (A) and (B) below, compute $\frac{dy}{dx}$.

A) $y = 5x \cot(2x^3)$.
B) $x \cos(y) = y^4$.

21. (10 points) Compute $\lim_{x \to +\infty} \left( 1 - \frac{5}{x} \right)^x$.

22. Find the absolute maximum and minimum values, if any, of $f(x) = 4x^3 - 3x^4$ on the interval $(-\infty, +\infty)$.

23. An open box is to be made from a 8-inch by 15-inch piece of cardboard by cutting squares of equal size from each of the four corners and bending up the sides. What size are the squares so that the box will have the largest volume possible?
24. On the axes provided on the next page, sketch the graph of the function \( f \) given below, on the grid on the following page, and identify the locations of all critical points, cusp, vertical tangent and inflection points. For your convenience \( f' \) and \( f'' \) are given. Label all intercepts and asymptotes, if any. Hint: \((-4)^{1/3}\) is about \(-1.6\). (10 points)

\[
\begin{align*}
  f(x) &= (x^2 - 4)^{\frac{1}{3}} \\
  f'(x) &= \frac{2x}{3(x^2 - 4)^{\frac{2}{3}}} \\
  f''(x) &= \frac{-2(x^2 + 12)}{9(x^2 - 4)^{5/3}}
\end{align*}
\]

25. Water runs into a inverted conical tank at the rate of 10 \( ft^3/min \). The tank has a height of 24 feet and a base with radius 6 ft. How fast is the water level rising when the depth of the water is 16 ft?

(Note: The formula for the volume of a cone is \( \frac{1}{3} \pi r^2 h \))