The following rules apply:

- **This is a closed-book exam.** You may not use any books or notes on this exam.

- **For free response questions, you must show all work.** Answers without proper justification will not receive full credit. Partial credit will be awarded for significant progress towards the correct answer. Cross off any work that you do not want graded.

- **For multiple choice questions, circle the letter of the best answer.** Make sure your circles include just one letter. These problems will be marked as correct or incorrect; partial credit will not be awarded for problems in this section.

- **You have 50 minutes to complete this exam.** When time is called, stop writing immediately and turn in your exam to the nearest proctor.

- **You may not use any electronic devices including (but not limited to) calculators, cell phone, or iPods.** Using such a device will be considered a violation of the university’s academic integrity policy and, at the very least, will result in a grade of 0 for this exam.
Part I: Free Response

1. (10 points) The trough pictured below is 15 feet long and 4 feet wide at the top. The ends of the trough are isosceles triangles with a height of 3 feet.

The trough is filled with liquid to a height of 1 foot. Set up an integral which represents the work required to pump all of the liquid out of the top of the trough. (The weight density of the liquid is 62.5 lb/ft³.) You do not have to evaluate your integral.

Hint: Use similar triangles.

\[ \text{Volume of a slice:} \quad V = \int_{0}^{3} \frac{4}{3} x (15) \, dx = 20x \, dx \]

\[ \text{Weight of slice:} \quad F = (62.5)(20x \, dx) \]

\[ \text{Displacement of slice:} \quad d = 3 - x \]

\[ W = \int_{0}^{3} (62.5)(20x)(3-x) \, dx \]
2. (15 points) Evaluate the following integral:

\[ \int \frac{x + 1}{x^2(x-1)} \, dx \]

**Partial Fraction Decomposition:**

\[ \frac{x + 1}{x^2(x-1)} = \frac{A}{x} + \frac{B}{x^2} + \frac{C}{x-1} \quad \Rightarrow \quad Ax(x-1) + B(x-1) + Cx^2 = x + 1 \]

**Solve for A, B, and C:**

\[
\begin{align*}
\text{At } x = 1: & \quad A(1)(0) + B(0) + C(1)^2 = 1 + 1 \\
& \quad C = 2
\end{align*}
\]

\[
\begin{align*}
\text{At } x = 0: & \quad A(0)(-1) + B(-1) + C(0)^2 = 1 \\
& \quad -B = 1 \\
& \quad B = -1
\end{align*}
\]

\[
\begin{align*}
Ax(x-1) + (-1)(x-1) + (2)x^2 &= x + 1 \\
A(x^2) - Ax - x + 1 + 2x^2 &= x + 1 \\
x^2\text{-Terms: } & \quad A + 2 = 0 \\
& \quad A = -2
\end{align*}
\]

\[
\int \frac{x + 1}{x^2(x-1)} \, dx = \int \left( \frac{-2}{x} - \frac{1}{x^2} + \frac{2}{x-1} \right) \, dx
\]

\[= -2 \ln|x| + \frac{1}{x} + 2 \ln|x-1| + C
\]

\[= 2 \ln \left| \frac{x-1}{x} \right| + \frac{1}{x} + C\]
3. (15 points) Solve the following initial value problem. You may leave your answer as an implicitly defined function.

\[
\begin{align*}
\frac{dy}{dx} &= e^{2x-y} \\
y(0) &= \ln 3
\end{align*}
\]

\[
\begin{align*}
\frac{dy}{dx} &= e^{2x} - y \\
\frac{dy}{dx} &= e^{2x} e^{-y} \\
y \, dy &= e^{2x} \, dx \\
\int e^y \, dy &= \int e^{2x} \, dx \\
e^y &= \frac{1}{2} e^{2x} + C \\
\ln^3 &= \frac{1}{2} e^{2(0)} + C \\
3 &= \frac{1}{2} + C \\
C &= \frac{5}{2} \\
e^y &= \frac{1}{2} e^{2x} + \frac{5}{2} \\
y &= \ln \left( \frac{1}{2} e^{2x} + \frac{5}{2} \right)
\end{align*}
\]
4. (15 points) Use integration by parts to evaluate the following integral:

\[ \int_{0}^{\pi/3} 6x \cos x \, dx \]

\[
\begin{aligned}
u &= 6x \\
dv &= \cos x \, dx \\
\frac{dv}{dx} &= \sin x \\
u &= 6x \\
du &= \sin x \, dx
\end{aligned}
\]

\[\int_{0}^{\pi/3} 6x \cos x \, dx = 6x \sin x \bigg|_{0}^{\pi/3} - \int_{0}^{\pi/3} 6 \sin x \, dx \]

\[= 6x \sin x \bigg|_{0}^{\pi/3} + 6 \cos x \bigg|_{0}^{\pi/3} \]

\[= 6 \left( \frac{\pi}{3} \right) \sin \left( \frac{\pi}{3} \right) - 6(0) \sin (0) + 6 \cos \left( \frac{\pi}{3} \right) - 6 \cos 0 \]

\[= 2\pi \left( \frac{\sqrt{3}}{2} \right) - 0 + 6 \left( \frac{1}{2} \right) - 6 \]

\[= \sqrt{3} \pi + 3 - 6 \]

\[= \sqrt{3} \pi - 3 \]
5. (15 points) Calculate the exact arc length of the curve \( y = \frac{4}{3} x^{3/2} + 1 \) over the interval \([0, 2]\)

\[
\frac{dy}{dx} = 2x^{1/2}
\]

\[
\left( \frac{dy}{dx} \right)^2 = 4x
\]

\[
L = \int_{0}^{2} \sqrt{1 + 4x} \, dx
\]

\[
= \frac{1}{4} \int_{1}^{9} \sqrt{u} \, du
\]

\[
= \frac{1}{4} \left( \frac{2}{3} \right) u^{3/2} \bigg|_{1}^{9}
\]

\[
= \frac{1}{6} \left( 9^{3/2} - 1^{3/2} \right)
\]

\[
= \frac{1}{6} \left( 27 - 1 \right)
\]

\[
= \frac{26}{6}
\]

\[
= \frac{13}{3}
\]
Part II: Multiple Choice

6. (5 points) Suppose $R$ is the region enclosed by $y = \sqrt{x}$, $x = 2$, and the $x$-axis. Which of the following is the volume that results from revolving $R$ around the $x$-axis?

(a) $\frac{1}{3}\pi$

(b) $\frac{1}{2}\pi$

(c) $\frac{2}{3}\pi$

(d) $\pi$

e) $2\pi$

7. (5 points) Which of the following is the partial fraction decomposition of $\frac{x^5 + 1}{x^4 + 3x^2 + 2}$?

(a) $\frac{A}{x^2 + 1} + \frac{B}{x^2 + 2}$

(b) $\frac{Ax + B}{x^2 + 1} + \frac{Cx + D}{x^2 + 2}$

(c) $x + \frac{A}{x^2 + 1} + \frac{C}{x^2 + 2}$

d) $x + \frac{Ax + B}{x^2 + 1} + \frac{Cx + D}{x^2 + 2}$

(e) $x^5 + \frac{Ax + B}{x^2 + 1} + \frac{Cx + D}{x^2 + 2}$
8. (5 points) Evaluate $\int_1^{\infty} \frac{1}{x^2} \, dx$

(a) $-2$

(b) $-1$

(c) 1

(d) 2

(e) The integral diverges.

9. (5 points) Evaluate $\int_0^1 \frac{1}{x^3} \, dx$

(a) $-2$

(b) $-1$

(c) 1

(d) 2

(e) The integral diverges.
10. (5 points) Consider the region $R$, shown below, which is enclosed by $y = x^2$, $y = 3x$ and $y = 4$.

Which of the following represents the volume of the solid which results from revolving $R$ around the line $x = 2$?

(a) $\pi \int_0^2 (x^2 - 3x)^2 \, dx$

(b) $\pi \int_0^{4/3} [(3x)^2 - (x^2)^2] \, dx + \pi \int_{4/3}^2 [(4)^2 - (x^2)^2] \, dx$

(c) $\pi \int_0^4 \left( \sqrt{y} - \frac{y}{3} \right)^2 \, dy$

(d) $\pi \int_0^4 \left[ (\sqrt{y})^2 - \left( \frac{y}{3} \right)^2 \right] \, dy$

(e) $\pi \int_0^4 \left[ \left( 2 - \frac{y}{3} \right)^2 - (2 - \sqrt{y})^2 \right] \, dy$
11. (5 points) Consider the region \( R \), shown below, which is enclosed by \( y = |x| \) and \( y = 3 \).

Which of the following represents the volume of the solid whose base is \( R \) and whose cross sections taken perpendicular to the \( y \)-axis are squares?

(a) \( \int_{-3}^{3} x^2 \, dx \)
(b) \( \int_{-3}^{3} (3 - |x|)^2 \, dx \)
(c) \( \int_{0}^{3} y^2 \, dy \)
(d) \( \int_{0}^{3} 4y^2 \, dy \)
(e) \( \int_{0}^{3} \frac{y^2}{4} \, dy \)