Math 122 - Exam 1 - 2/2/2015

Name: ________________________________  Section: ______

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The following rules apply:

- **This is a closed-book exam.** You may *not* use any books or notes on this exam.

- **For free response questions, you must show all work.** Answers without proper justification will not receive full credit. Partial credit will be awarded for significant progress towards the correct answer. Cross off any work that you do not want graded.

- **For multiple choice questions, circle the letter of the best answer.** Make sure your circles include just one letter. These problems will be marked as correct or incorrect; partial credit will not be awarded for problems in this section.

- **You have 50 minutes to complete this exam.** When time is called, stop writing immediately and turn in your exam to the nearest proctor.

- **You may not use any electronic devices including (but not limited to) calculators, cell phone, or iPods.** Using such a device will be considered a violation of the university’s academic integrity policy and, at the very least, will result in a grade of 0 for this exam.

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Part I: Free Response

**Summation Formulas:**

\[
\sum_{k=1}^{n} k = \frac{n(n + 1)}{2} \quad \sum_{k=1}^{n} k^2 = \frac{n(n + 1)(2n + 1)}{6} \quad \sum_{k=1}^{n} k^3 = \left( \frac{n(n + 1)}{2} \right)^2
\]

1. (25 points) Suppose \( f(x) = x^2 - 1 \).

   (a) Using \( n \) rectangles of equal width and choosing \( x_k^* \) to be the right endpoint of the \( k \)th subinterval, estimate the net signed area between the graph of \( f(x) \) and the \( x \)-axis on the interval \([1, 4]\). Express your answer in “closed form” by applying appropriate summation formula(s).
(b) Using part (a) and an appropriate limit, calculate the exact net signed area between the graph of $f(x)$ and the $x$-axis on the interval $[1, 4]$.

(c) Use an appropriate definite integral and the Fundamental Theorem of Calculus to verify your answer from part (b).
2. (15 points) Evaluate the following integral:

\[ \int_{1}^{2} \frac{\sin(\pi/x)}{x^2} \, dx \]
3. Suppose $R$ is the region in the first quadrant which is enclosed by the graphs of $y = \frac{1}{x}$, $y = 1$ and $x = e^2$.
   
   (a) (6 points) Sketch $R$ on the axes provided. Label all curves and shade $R$.

   (b) (14 points) Compute the area of $R$. 
4. (15 points) Evaluate the following indefinite integrals:

(a) \( \int (x + 1)\sqrt{x} \, dx \)

(b) \( \int x\sqrt{x - 1} \, dx \)
Part II: Multiple Choice

5. (5 points) Suppose that the area between the graph of \( f(x) = x^2 \) and the interval \([-4, -1]\) is approximated using three rectangles of equal width and left endpoints. This approximation is...

(a) an overestimate because \( f(x) \) is increasing on \([-4, -1]\).

(b) an overestimate because \( f(x) \) is decreasing on \([-4, -1]\).

(c) an underestimate because \( f(x) \) is increasing on \([-4, -1]\).

(d) an underestimate because \( f(x) \) is decreasing on \([-4, -1]\).

(e) equal to the exact area between the graph \( f(x) \) and \([-4, -1]\).

6. (5 points) Suppose \( f(x) = \begin{cases} 
  x - 2 & \text{if } x \leq 4 \\
  2 & \text{if } x > 4
\end{cases} \). Evaluate \( \int_{0}^{6} f(x) \, dx \).

(a) 0

(b) 2

(c) 4

(d) 6

(e) 8
7. (5 points) Suppose \( F(x) = \int_0^x f(t) \, dt \), where \( f(t) \) is as shown below.

Which of the following statements is/are true?

I. \( F(1) > F(0) \)

II. \( F(5) > F(0) \)

III. \( F(5) > F(1) \)

(a) I only
(b) II only
(c) I and II only
(d) II and III only
(e) I, II, and III

8. (5 points) Evaluate \( \int_{-1}^{1} \sqrt{1 - x^2} \, dx \).

(a) 0
(b) 2
(c) \( \frac{\pi}{4} \)
(d) \( \frac{\pi}{2} \)
(e) \( \pi \)
9. (5 points) Consider the region \( R \), shown below, which is enclosed by \( y = x^2 \), \( y = 3x \), and \( y = 4 \).

Which of the following represents the area of \( R \)?

(a) \( \int_{0}^{2} (3x - x^2) \, dx \)
(b) \( \int_{0}^{2} (3x - x^2) \, dx \)
(c) \( \int_{0}^{4} (x^2 - 3x) \, dx \)
(d) \( \int_{0}^{4} (\sqrt{y} - \frac{y}{3}) \, dy \)
(e) \( \int_{0}^{4} (y^2 - 3y) \, dy \)