Local Linear Approximation

SUGGESTED REFERENCE MATERIAL:

As you work through the problems listed below, you should reference Chapter 13.4 of the recommended textbook (or the equivalent chapter in your alternative textbook/online resource) and your lecture notes.

EXPECTED SKILLS:

- Be able to compute the local linear approximation for a function of two or more variables at a given point.
- Be able to use a local linear approximation to estimate a given quantity.

PRACTICE PROBLEMS:

For problems 1-5, find the local linear approximation $L(x, y)$ of the given function at the specified point.

1. $f(x, y) = x^2 - y^2; \ P(1, 2)$
   
   $L(x, y) = 3 + 2x - 4y$

2. $f(x, y) = \frac{x + y}{x - y}; \ P(2, 1)$
   
   $L(x, y) = 3 - 2x + 4y$

3. $f(x, y) = e^x \sin y; \ P\left(\ln 3, \frac{\pi}{2}\right)$
   
   $L(x, y) = 3 - 3\ln 3 + 3x$

4. $f(x, y) = \ln (x^2 - y^2); \ P\left(2, \sqrt{3}\right)$
   
   $L(x, y) = -2 + 4x - 2\sqrt{3}y$

5. $f(x, y) = \tan^{-1}\left(\frac{x}{y}\right); \ P(1, 1)$
   
   $L(x, y) = \frac{\pi}{4} + \frac{1}{2}x - \frac{1}{2}y$
6. Find the local linear approximation of the function $f(x, y) = \sqrt{32 - 3x^2 - y^2}$ at $(1, 2)$ and use it to approximate $f(0.98, 2.01)$.

\[
f(x, y) \approx L(x, y) = \frac{32}{5} - \frac{3}{5}x - \frac{2}{5}y \text{ for } (x, y) \text{ near } (1, 2).
\]

So, $f(0.98, 2.01) \approx L(0.98, 2.01) = \frac{626}{125}$

7. Suppose that $f(x, y)$ is a differentiable function at the point $(2, 3)$ with $f(2, 3) = 1$, $f_x(2, 3) = 5$, and $f_y(2, 3) = -2$. Estimate $f(1.98, 3.01)$.

\[
f(x, y) \approx L(x, y) = -3 + 5x - 2y \text{ for } (x, y) \text{ near } (2, 3).
\]

So, $f(1.98, 3.01) \approx L(1.98, 3.01) = 0.88$

8. Find the local linear approximation $L(x, y, z)$ to $f(x, y, z) = 3x^2 - 2y^2 + xz^3$ at the point $P(-1, 2, 1)$.

\[
L(x, y, z) = 8 - 5x - 8y - 3z
\]

9. Verify that $e^x \cos y \approx 1 + x$ for $(x, y)$ near $(0, 0)$.

Show that the local linear approximation to $f(x, y) = e^x \cos y$ at the point $(0, 0)$ is $L(x, y) = 1 + x$.

10. Verify that $(x + y)^3 \approx -16 + 12x + 12y$ for $(x, y)$ near $(1, 1)$.

Show that the local linear approximation to $f(x, y) = (x + y)^3$ at the point $(1, 1)$ is $L(x, y) = -16 + 12x + 12y$.

11. At a particular point $P(x_0, y_0)$, the local linear approximation of $f(x, y) = xy + y^2$ is $L(x, y) = -15 + 3x + 8y$. What is the point $P$?

$(2, 3)$