Math 200 - Exam 2 - 5/30/2014

NAME: Solutions

SECTION: _______________________

Directions:

- For the free response section, you must show all work. Answers without proper justification will not receive full credit. Partial credit will be awarded for significant progress towards the correct answer. Cross off any work that you do not want graded.

- For word problems, all answers must include appropriate units in order to earn full credit.

- You have 50 minutes to complete this exam. When time is called, STOP WRITING IMMEDIATELY.

- You may not use any electronic devices including (but not limited to) calculators, cell phones, or iPods. Using such a device will be considered a violation of the university’s academic integrity policy and, at the very least, will result in a grade of 0 for the exam.

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Free Response

Reminder: For each of the following problems, you must show all of your work to earn full credit. Simplify all answers.

1. Consider $f(x, y) = 2x^3 + y^2 - 9x^2 - 4y + 12x - 2$.

   (a) Identify all critical points of $f(x, y)$.

   (b) Classify each critical point from part (a) as the location of a relative (local) maximum, relative (local) minimum, or saddle point.

   \[
   \begin{align*}
   f_x &= 6x^2 - 18x + 12 \\
   f_y &= 2y - 4 \\
   6x^2 - 18x + 12 &= 0 \\
   6(x^2 - 3x + 2) &= 0 \\
   6(x-1)(x-2) &= 0 \\
   x &= 1, 2 \\
   
   \text{CRITICAL POINTS: (1, 2); (2, 2)}
   \end{align*}
   \]

   \[
   \begin{align*}
   f_{xx} &= 12x - 18; \\
   f_{yy} &= 2; \\
   f_{xy} &= 0
   \end{align*}
   \]

   \[
   \begin{align*}
   D(1, 2) &= (-6)(2) - 0 = -12 < 0 \\
   \text{saddle point at (1, 2)}
   \end{align*}
   \]

   \[
   \begin{align*}
   D(2, 2) &= (6)(2) - 0 = 12 > 0 \\
   f_{xx}(2, 2) &= 6 > 0 \\
   \text{relative minimum at (2, 2)}
   \end{align*}
   \]
2. Find all points on the ellipsoid \( x^2 + 2y^2 + 3z^2 = 12 \) where the tangent plane is perpendicular to the line \( L \), defined below.

\[
L: \begin{cases} 
  x = 1 + 2t \\
  y = 3 + 4t \\
  z = 2 - 6t
\end{cases}
\]

\[ \vec{n} = \langle 2, 4, -6 \rangle \]

\[ F(x, y, z) = x^2 + 2y^2 + 3z^2 \]

\[ \nabla F(x, y, z) = \langle 2x, 4y, 6z \rangle \]

If \( \nabla F \parallel \vec{n} \), then for some \( k \), \( \langle 2x, 4y, 6z \rangle = k \langle 2, 4, -6 \rangle \)

\[
\begin{align*}
2x &= 2k \\
4y &= 4k \\
6z &= -6k
\end{align*}
\]

\[
\begin{align*}
x &= k \\
y &= k \\
z &= -k
\end{align*}
\]

\[
(k)^2 + 2(k)^2 + 3(-k)^2 = 12
\]

\[
k^2 + 2k^2 + 3k^2 = 12
\]

\[
6k^2 = 12
\]

\[
k = \pm \sqrt{2}
\]

\[ \langle \sqrt{2}, \sqrt{2}, -\sqrt{2} \rangle \text{ and } \langle -\sqrt{2}, -\sqrt{2}, \sqrt{2} \rangle \]
3. Suppose $R$ is the region in the first quadrant enclosed by $y = x$, $y = \frac{\pi}{2}$, and $x = 0$.

Evaluate $\iint_R \cos y \, dA$.

\[
\iint_R \cos y \, dA = \int_0^\pi \int_0^\frac{\pi}{2} \cos y \, dy \, dx
\]

\[
= \int_0^\pi \left[ \sin y \right]_0^{\frac{\pi}{2}} \, dx
\]

\[
= \int_0^\pi (1 - 0) \, dx
\]

\[
= \left[ x + \cos x \right]_0^\pi
\]

\[
= \left( \frac{\pi}{2} + 0 \right) - \left( 0 + 1 \right)
\]

\[
= \frac{\pi}{2} - 1
\]

OR:

\[
\iint_R \cos y \, dA = \int_0^\frac{\pi}{2} \int_0^y \cos y \, dx \, dy
\]

\[
= \int_0^\frac{\pi}{2} \left[ x \cos y \right]_0^y \, dy
\]

\[
= \int_0^\frac{\pi}{2} y \cos y \, dy
\]

\[u = y; \ dv = \cos y; \ du = dy; \ v = \sin y\]

\[
= \left[ y \sin y \right]_0^\frac{\pi}{2} - \int_0^\frac{\pi}{2} \sin y \, dy
\]

\[
= \frac{\pi}{2} + \cos y \bigg|_0^{\frac{\pi}{2}}
\]

\[
= \frac{\pi}{2} + (0 - 1)
\]

\[
= \frac{\pi}{2} - 1
\]
4. Let $R = \{(x, y) : 4 \leq x^2 + y^2 \leq 16 \text{ and } y \geq x\}$.

(a) Sketch $R$ on the axes provided.

(b) Evaluate $\iint_R \frac{1}{\sqrt{x^2 + y^2}} \, dA$ using polar coordinates.

\[
\iint_R \frac{1}{\sqrt{x^2 + y^2}} \, dA = \int_0^{\pi/4} \int_2^4 r \, dr \, d\theta
\]

\[
= \int_2^4 \int_0^{\pi/4} r \, dr \, d\theta
\]

\[
= \int_2^4 \left[ \frac{r^2}{2} \right]_0^{\pi/4} \, d\theta
\]

\[
= \int_2^4 \left( \frac{\pi}{8} - 1 \right) \, d\theta
\]

\[
= 2 \theta \left[ \frac{\pi}{8} - 1 \right]_2^4
\]

\[
= 2 \left( \frac{5\pi}{8} - \frac{\pi}{4} \right)
\]

\[
= \frac{2 \pi}{8}
\]
5. Let $f(x, y) = xe^{2y}$

(a) Compute the directional derivative of $f(x, y)$ at $P(-2, 0)$ in the direction of $\vec{u}$, a unit vector which makes an angle of $\frac{\pi}{3}$ with the positive $x$ axis.

(b) In which direction is $f(x, y)$ increasing most rapidly at $P(-2, 0)$? Give your answer as a unit vector.

\[
\nabla f = \begin{pmatrix} e^{2y} \\ 2xe^{2y} \end{pmatrix} \\
\nabla f(-2, 0) = \begin{pmatrix} 1 \\ -4 \end{pmatrix} \\
\vec{u} = \begin{pmatrix} \cos \frac{\pi}{3} \\ \sin \frac{\pi}{3} \end{pmatrix} \\
\vec{u} = \begin{pmatrix} \frac{1}{2} \\ \frac{\sqrt{3}}{2} \end{pmatrix} \\
D_\vec{u} f(-2, 0) = \begin{pmatrix} 1 \\ -4 \end{pmatrix} \cdot \begin{pmatrix} \frac{1}{2} \\ \frac{\sqrt{3}}{2} \end{pmatrix} = \sqrt{3} \\

\|
\nabla f(-2, 0)\| = \sqrt{1^2 + 4^2} = \sqrt{17} \\
\vec{u} = \begin{pmatrix} \frac{1}{\sqrt{17}} \\ -\frac{4}{\sqrt{17}} \end{pmatrix}
\]

6
Multiple Choice

Circle the letter of the best answer. Make sure your circles include just one letter. These problems will be marked as correct or incorrect; partial credit will not be awarded for problems in this section. All questions are worth 5 points.

6. The graph shown below depicts some level curves of a differentiable function $f(x, y)$.

Which of the vectors is most likely to be $\nabla f$ at $P$?

(a) $\vec{a}$

(b) $\vec{b}$

(c) $\vec{c}$

(d) $\vec{d}$

(e) $\vec{e}$
7. Which of the following is the local linear approximation (tangent plane approximation) of \( z = x^2 - y^2 \) at \( P(2,1) \)?

(a) \( L(x, y) = 1(x - 2) + 2(y - 1) + 3 \)

(b) \( L(x, y) = 2(x - 2) - 1(y - 1) + 3 \)

(c) \( L(x, y) = 2(x - 2) + 1(y - 1) + 3 \)

(d) \( L(x, y) = 4(x - 2) - 2(y - 1) + 3 \)

(e) \( L(x, y) = 4(x - 2) - 4(y - 1) + 3 \)

8. Suppose \( z = f(x, y) \) is a differentiable function of \( x \) and \( y \), with \( x = 3u - v \) and \( y = u^2 + v \). And, consider the following table of values:

<table>
<thead>
<tr>
<th>((x,y))</th>
<th>(f(x,y))</th>
<th>(f_x(x,y))</th>
<th>(f_y(x,y))</th>
</tr>
</thead>
<tbody>
<tr>
<td>((2,-1))</td>
<td>6</td>
<td>1</td>
<td>9</td>
</tr>
<tr>
<td>((7,3))</td>
<td>4</td>
<td>-3</td>
<td>5</td>
</tr>
</tbody>
</table>

Which of the following is \( \frac{\partial z}{\partial u} \big|_{(u,v)=(2,-1)} \)?

(a) 8

(b) 11

(c) 12

(d) 25

(e) 39
9. Suppose \( f(x, y) \) is an integrable function. Which of the following integrals results from reversing the order of integration of \( \int_{0}^{1} \int_{0}^{3x+1} f(x, y) \, dy \, dx \)?

(a) \( \int_{0}^{4} \int_{0}^{1} f(x, y) \, dx \, dy \)

(b) \( \int_{0}^{3x+1} \int_{0}^{1} f(x, y) \, dx \, dy \)

(c) \( \int_{0}^{4} \int_{\frac{1}{3}y - \frac{1}{3}}^{1} f(x, y) \, dx \, dy \)

(d) \( \int_{0}^{1} \int_{0}^{1} f(x, y) \, dx \, dy + \int_{1}^{4} \int_{0}^{1} f(x, y) \, dx \, dy \)

(e) \( \int_{0}^{1} \int_{0}^{1} f(x, y) \, dx \, dy + \int_{1}^{4} \int_{\frac{1}{3}y - \frac{1}{3}}^{1} f(x, y) \, dx \, dy \)