
NAME: ________________

SECTION: ________________________

Directions:

- For the free response section, you must show all work. Answers without proper justification will not receive full credit. Partial credit will be awarded for significant progress towards the correct answer. Cross off any work that you do not want graded.

- For word problems, all answers must include appropriate units in order to earn full credit.

- You have 50 minutes to complete this exam. When time is called, STOP WRITING IMMEDIATELY.

- You may not use any electronic devices including (but not limited to) calculators, cell phones, or iPods. Using such a device will be considered a violation of the university’s academic integrity policy and, at the very least, will result in a grade of 0 for the exam.

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Free Response

Reminder: For each of the following problems, you must show all of your work to earn full credit. Simplify all answers.

1. Find the acute angle between the planes which are tangent to $S_1 : z = x^2 + y^2$ and $S_2 : 4x^2 + y^2 + z^2 = 9$ at the point $(-1, 1, 2)$.

Let $f(x, y, z) = x^2 + y^2 - z$

$\nabla f(x, y, z) = \langle 2x, 2y, -1 \rangle$

$\nabla f(-1, 1, 2) = \langle -2, 2, -1 \rangle$

$\overrightarrow{n}_1$

Let $g(x, y, z) = 4x^2 + y^2 + z^2$

$\nabla g(x, y, z) = \langle 8x, 2y, 2z \rangle$

$\nabla g(-1, 1, 2) = \langle -8, 2, 4 \rangle$

$\overrightarrow{n}_2$

$\overrightarrow{n}_1 \cdot \overrightarrow{n}_2 = ||\overrightarrow{n}_1|| \ ||\overrightarrow{n}_2|| \cos \theta$

$\overrightarrow{n}_1 \cdot \overrightarrow{n}_2 = (-2)(-8) + (2)(2) + (-1)(4) = 16$

$||\overrightarrow{n}_1|| = \sqrt{(-2)^2 + (2)^2 + (-1)^2} = 3$

$||\overrightarrow{n}_2|| = \sqrt{(-8)^2 + (2)^2 + (4)^2} = \sqrt{84}$

Thus,

$16 = 3\sqrt{84} \cos \theta$

$\theta = \cos^{-1} \left( \frac{16}{3\sqrt{84}} \right)$
2. Find the absolute extrema (maximum and minimum values) of \( f(x, y) = 1 - 6x + 3y \) on \( R = \{(x, y) : x^2 \leq y \leq 4\} \), shown below.

**Critical Points**

\[
\begin{align*}
\frac{\partial f}{\partial x} &= -6 
eq 0 \\
\frac{\partial f}{\partial y} &= 3 
eq 0
\end{align*}
\]

\( \therefore \) no critical pts.

**Boundaries**

\[
\begin{align*}
\begin{cases}
 x = t \\
y = 4
\end{cases}
\quad &-2 \leq t \leq 2
\end{align*}
\]

\( f(t, 4) = 1 - 6t + 12 
\]

\( \therefore g(t) = 13 - 6t \quad [-2, 2] \)

\( \text{endpts: } (-2, 4) \quad (2, 4) \)

\( \therefore g'(t) = -6 \neq 0 \quad \therefore \text{none} \)

**Second Order Test**

\[
\begin{align*}
\begin{cases}
 x = t \\
y = t^2
\end{cases}
\quad &-2 \leq t \leq 2
\end{align*}
\]

\[
 f(t, t^2) = 1 - 6t + 3t^2 = h(t)
\]

\( [-2, 2] \)

\( \text{endpts: } (-2, 4) \quad (2, 4) \)

\( \therefore h'(t) = -6 + 6t = 0 
\]

\( t = 1 
\]

\( \therefore (1, 1) \)

\[
\begin{array}{c|c|c}
(x, y) & f(x, y) \\
\hline
(1, 1) & -2 \\
(2, 4) & 1 \\
(-2, 4) & 25 \\
\end{array}
\]

\( \therefore \text{max} \text{ of 25 at } (-2, 4) \)

\( \min \text{ of -2 at } (1, 1) \)
3. Consider the solid which is bounded above by the hemisphere \( z = \sqrt{4 - x^2 - y^2} \), bounded below by the \( xy \) plane, and is outside of the cylinder \( x^2 + y^2 = 1 \).

Set up a double integral and polar coordinates which, when solved, gives the volume of this solid. **DO NOT EVALUATE YOUR INTEGRAL(S).**

\[
V = \iint_R \sqrt{4 - x^2 - y^2} - 0 \, dA
\]

\[
= \int_0^{2\pi} \int_0^2 \sqrt{4 - r^2} \, r \, dr \, d\theta
\]
4. Calculate the distance from the point $P(1,1,1)$ to the line which passes through $A(0,2,3)$ and $B(1,2,1)$.

\[ \vec{AP} = \langle 1-0, 1-2, 1-3 \rangle = \langle 1, -1, -2 \rangle \]

\[ \vec{AB} = \langle 1-0, 2-2, 1-3 \rangle = \langle 1, 0, -2 \rangle \]

\[ \left\| \text{proj}_{\vec{AB}} \vec{AP} \right\| = \frac{\vec{AP} \cdot \vec{AB}}{\| \vec{AB} \|} = \frac{1+0+4}{\sqrt{1+0+4}} = \frac{5}{\sqrt{5}} = \sqrt{5} \]

\[ \| \vec{AP} \| = \sqrt{1+1+4} = \sqrt{6} \]

\[ (\sqrt{5})^2 + (d)^2 = (\sqrt{6})^2 \]

\[ 5 + d = 6 \]

\[ d = 1 \]
5. A particle moves through 3-space in such a way that its velocity is \( \mathbf{v}(t) = (2t, e^{t/2}, 3\sqrt{t+4}) \). The particle's initial position at time \( t = 0 \) is \( (1, 2, 3) \). Set up and solve an initial value problem which gives \( \mathbf{r}(t) \), the position of the particle at time \( t \).

\[
\frac{d\mathbf{r}}{dt} = (2t, e^{t/2}, 3\sqrt{t+4})
\]

\[
\mathbf{r}(0) = (1, 2, 3)
\]

\[
\mathbf{r}(t) = \int (2t, e^{t/2}, 3\sqrt{t+4}) \, dt
\]

\[
\mathbf{r}(t) = (t^2, 2e^{t/2}, 2(t+4)^{3/2}) + \mathbf{C}
\]

\[
\mathbf{r}(0) = (0, 2, 2(4)^{3/2}) + \mathbf{C} = (1, 2, 13)
\]

\[
(0, 1, 16) + \mathbf{C} = (1, 2, 13)
\]

\[
\mathbf{C} = (1, 0, -13)
\]

So,

\[
\mathbf{r}(t) = (t^2+1, 2e^{t/2}, 2(t+4)^{3/2} - 13)
\]
6. Calculate an equation for the line of intersection of planes $P_1 : x + y + z = 12$ and $P_2 : 3x - y + 2z = 4$. Hint: The point $(4, 8, 0)$ is on both planes.

\[ \vec{L}(t) = \vec{L}_0 + t\vec{v} \]

**Given** $\vec{L}_0 = \langle 4, 8, 0 \rangle$

To find $\vec{v}$, notice $\vec{n}_1 = \langle 1, 1, 1 \rangle$

$\vec{n}_2 = \langle 3, -1, 2 \rangle$

\[ \vec{v} = \vec{n}_1 \times \vec{n}_2 \]

\[ = \begin{vmatrix}
\mathbf{i} & \mathbf{j} & \mathbf{k} \\
1 & 1 & 1 \\
3 & -1 & 2
\end{vmatrix} \\
= \langle 3, 1, -4 \rangle \]

So \[ \vec{L}(t) = \langle 3, 1, -4 \rangle t + \langle 4, 8, 0 \rangle \]
7. Let \( R \) be the region enclosed by \( y = x + 4 \), \( y = 4 - 2x \), and the \( x \)-axis,

(a) Set up \( \iint_{R} f(x, y) \, dA \) with the order of integration as \( dy \, dx \).

(b) Set up \( \iint_{R} f(x, y) \, dA \) with the order of integration as \( dx \, dy \).

\[\begin{align*}
\text{(a)} & \quad \int_{-4}^{0} \int_{0}^{x+4} f(x, y) \, dy \, dx + \int_{0}^{2} \int_{0}^{4-2x} f(x, y) \, dy \, dx \\
\text{(b)} & \quad \int_{0}^{y-4} \int_{0}^{\frac{y}{2}} f(x, y) \, dx \, dy
\end{align*}\]