The following rules apply:

- **This is a closed-book exam.** You may *not* use any books or notes on this exam.

- **For free response questions, you must show all work.** Answers without proper justification will not receive full credit. Partial credit will be awarded for significant progress towards the correct answer. Cross off any work that you do not want graded.

- **For multiple choice questions, circle the letter of the best answer.** Make sure your circles include just one letter. These problems will be marked as correct or incorrect; partial credit will not be awarded for problems in this section.

- **You have 50 minutes to complete this exam.** When time is called, stop writing immediately and turn in your exam to the nearest proctor.

- **You may not use any electronic devices including (but not limited to) calculators, cell phone, or iPods.** Using such a device will be considered a violation of the university’s academic integrity policy and, at the very least, will result in a grade of 0 for this exam.
Part I: Free Response

1. (15 points) Consider the lines

\[ L_1 : \begin{cases} x &= 1 + 2t_1 \\ y &= 2 + 3t_1 \\ z &= 3 + 4t_1 \end{cases} \quad L_2 : \begin{cases} x &= 2 + 2t_2 \\ y &= 4 + 2t_2 \\ z &= -1 - 4t_2 \end{cases} \]

(a) Determine the point of intersection of the two lines.

(b) Find an equation for the plane containing \( L_1 \) and \( L_2 \).
2. (15 points) The curves \( \vec{r}_1(t) = \langle t, t^2 + 1, t^3 \rangle \) and \( \vec{r}_2(t) = \langle 3 \sin t, \cos t, 3t \rangle \) intersect at the point \( P(0, 1, 0) \).

(a) Find the angle between the tangent vectors to the curves at \( P \).

(b) Find parametric equations for the line tangent to \( \vec{r}_2(t) \) at \( P \).
3. (10 points) Find a vector valued function $\mathbf{r}(t)$ which satisfies the following:

\[
\begin{aligned}
\mathbf{r}(0) &= \langle 3, -2, 1 \rangle \\
\frac{d\mathbf{r}}{dt} &= \langle 5 \sin(5t), \cos(2t), 2t \rangle
\end{aligned}
\]
Part II: True/False

4. (12 points) For each of the following, circle T if the statement is true and F if the statement is false.

T F Suppose $\vec{v}$ and $\vec{w}$ are vectors in 3-space. It is possible that $\vec{v} \cdot \vec{w} > 0$ and $||\vec{v} \times \vec{w}|| = 0$.

T F Suppose $\vec{v}$ and $\vec{w}$ are both unit vectors in 3-space. Then, $\vec{v} \times \vec{w}$ is also a unit vector.

T F The plane $y = 3$ is perpendicular to the $xz$-plane.

T F Suppose $\vec{v}$ and $\vec{w}$ are non-zero vectors in 3-space. The projection vector $\text{Proj}_{\vec{w}} \vec{v}$ is parallel to $\vec{w}$.
Part III: Matching

5. (8 points) For each of the following, write the number of the graphed surface which best represents the given equation.

(a) \[ x^2 + y^2 - z^2 = 1 \]

(b) \[ x^2 - y^2 + z = 0 \]

(c) \[ x^2 + y^2 - z^2 = 0 \]

(d) \[ -x^2 - y^2 + z^2 = 1 \]
Part IV: Multiple Choice

6. (5 points) Consider the figure shown below.

Which of the following is the component form of $\overrightarrow{AB} + \overrightarrow{BC} + \overrightarrow{CD}$?

(a) $\langle 3, -6 \rangle$

(b) $\langle 2, -5 \rangle$

(c) $\langle 0, 4 \rangle$

(d) $\langle -2, -2 \rangle$

(e) $\langle -5, -4 \rangle$

7. (5 points) The magnitude of the vector $\langle 2, 9, -6 \rangle$ is:

(a) 1

(b) 5

(c) 7

(d) 11

(e) 121
8. (5 points) The dot product $\langle 3, 3, -1 \rangle \cdot \langle 1, 0, -2 \rangle$ is:

(a) $\langle 4, 3, -3 \rangle$

(b) $\langle 3, 0, 2 \rangle$

(c) 0

(d) 4

(e) 5

9. (5 points) Which of the following vectors is parallel to the line $L_1$:

\[
\begin{align*}
x &= 1 - 4t \\
y &= 3t \\
z &= -7
\end{align*}
\]

(a) $\langle 1, 3, -7 \rangle$

(b) $\langle -1, 3, 0 \rangle$

(c) $\langle -3, 3, -7 \rangle$

(d) $\langle 8, -6, 0 \rangle$

(e) $\langle 2, 0, -14 \rangle$
10. (5 points) Find the distance from the point $P(1, 1, 3)$ to the plane $3x + 2y + 6z = 6$.

(a) 0

(b) 11/7

(c) 17/7

(d) 12/5

(e) 3

11. (5 points) Which of the following statements is true for all three-dimensional vectors $\vec{u}$, $\vec{v}$, and $\vec{w}$, if $\theta$ is the angle between $\vec{u}$ and $\vec{v}$?

I. $\vec{u} \times \vec{v} = \vec{v} \times \vec{u}$

II. $\vec{u} \cdot (\vec{v} \times \vec{w}) = (\vec{v} \times \vec{w}) \cdot \vec{u}$

III. $||\vec{v} \times \vec{w}|| = ||\vec{v}|| ||\vec{w}|| \cos \theta$

IV. $(\vec{u} \times \vec{v}) \cdot \vec{u} = 0$

(a) I and II only

(b) II and IV only

(c) III and IV only

(d) I, II, and III only

(e) All four statements are true.
12. (5 points) Which of the following points is on both the curve 
\( \vec{\ell}(t) = (2 - t, 3 + 2t, 4 + t) \) and the surface \( 2x + 3y + 4z = 21 \)?

(a) \((1, 1, 1)\)

(b) \((4, 3, 2)\)

(c) \((2, 3, 2)\)

(d) \((3, 1, 3)\)

(e) \((2, 2, 2)\)

13. (5 points) Determine the center and radius of the sphere \( x^2 + y^2 + z^2 - 6x - 16y + 20z = -92 \)

(a) Center: \((3, 8, -10)\), Radius: 9

(b) Center: \((3, 8, -10)\), Radius: 81

(c) Center: \((3, 8, 10)\), Radius: 9

(d) Center: \((-3, -8, 10)\), Radius: 9

(e) Center: \((-3, -8, 10)\), Radius: 81