Show all your work on the exam paper, legibly and in detail, to receive full credit. The use of a calculator or any other electronic device is prohibited. You may only use techniques discussed to date in class. You must simplify all answers unless you are explicitly instructed not to.

1. (a) (9 pts) Find the partial fraction expansion of the following rational function:

\[
\frac{x^2 - 8}{x^3 - x}
\]

(b) (8 pts) Evaluate the following indefinite integral:

\[
\int \frac{x^2 - 8}{x^3 - x} \, dx
\]
2. (12 pts) What is the partial fraction decomposition for the following rational function? Do NOT try to evaluate the unknown constants in the partial fraction decomposition.

\[
\frac{5 + 2x}{(x^2 + 1)x^3(x + 1)(x^2 + 4)^3}
\]
3. You are given that the partial fraction decomposition of:

$$\frac{6x^3+3x^2+2x-7}{(x^2+1)(x-1)(x+1)}$$

is

$$\frac{2x+A}{x^2+1} + \frac{1}{x-1} + \frac{3}{x+1}$$

a. (5 pts) Find the numerical value of $A$.

b. (5 pts) Evaluate the integral

$$\int \frac{6x^3+3x^2+2x-7}{(x^2+1)(x-1)(x+1)} \, dx$$
4.  a)(10 pts) Find the general solution to the following differential equation in implicit form.

\[ \frac{dy}{dx} - e^{-y} \sec^2(x) = 0 \]

b)(6 pts) Put the solution from part a) into explicit form.
5. (a) (4 pts) Using integration by parts, compute the following indefinite integral:

\[ \int x e^x \, dx \]

(b) (12 pts) Find the general solution to the following differential equation. Express the family of solutions as explicit functions of \( x \).

\[ y'(x) + \frac{1}{x} y(x) = e^x , \quad x > 0 \]
6. (a)(10 pts) At time $t = 0$, a tank contains 7 pounds of salt dissolved in 100 gallons of water. Then salt water containing 3 pounds of salt per gallon enters the tanks at a rate of $8 \text{ gal/min}$, and the mixed solution is drained from the tank at the rate of $8 \text{ gal/min}$. Let $S = S(t)$ be the amount of salt in the tank at time $t$. Write the differential equation, and initial condition, whose solution is $S = S(t)$. Do NOT solve the differential equation.

(b)(7 pts) At time $t = 0$, a tank contains 7 pounds of salt dissolved in 100 gallons of water. Then salt water containing 3 pounds of salt per gallon enters the tanks at a rate of $5 \text{ gal/min}$, and the mixed solution is drained from the tank at the rate of $8 \text{ gal/min}$. Let $S = S(t)$ be the amount of salt in the tank at time $t$. Write the differential equation, and initial condition, whose solution is $S = S(t)$. Do NOT solve the differential equation.
7. (12 pts) A scientist wants to determine the half-life of a certain radioactive substance. She determines that in exactly 5 days a 80 milligram sample decays to 10 milligrams. Based on this data, what is the half-life?