Sample Final Exam: Solutions

Problem 9. Suppose that every student in a discrete mathematics class of 25 students is a freshman, a sophomore, or a junior.

(a) Show that there are at least 9 freshman, at least 9 sophomores, or at least 9 juniors in the class.

(b) Show that there are either at least 3 freshman, at least 19 sophomores, or at least 5 juniors in the class.

Solution. (a) If there are less than or equal 8 freshmen, less than or equal 8 sophomores, and less than or equal 8 juniors in the class, then altogether there are no more than 24 students in the class, which is not the case. Therefore, our assumption is wrong, and there are at least 9 freshman, at least 9 sophomores, or at least 9 juniors in the class.

(b) If there are less than or equal 2 freshmen, less than or equal 18 sophomores, and less than or equal 4 juniors in the class, then altogether there are no more than 24 students in the class, which is not the case. Therefore, our assumption is wrong, and there are either at least 3 freshmen, at least 19 sophomores, or at least 5 juniors in the class.

Problem 10. Thirteen people on a softball team show up for a game.

(a) How many ways are there to choose 10 players to take the field?

(b) How many ways are there to assign the 10 positions by selecting players from the 13 people who show up?

(c) Of the 13 people who show up, 3 are women. How many ways are there to choose 10 players to take the field if at least one of these players must be a woman?

Solution. (a) \[ C(13, 10) = \frac{13!}{10!3!} = 13 \cdot 12 \cdot 11 = 286. \]

(b) \[ P(13, 10) = \frac{13!}{(13-10)!} = 13 \cdot 11 \cdot 10 \cdot 9 \cdot 8 \cdot 7 \cdot 6 \cdot 5 \cdot 4. \]

(c) If there is exactly one woman chosen, this is possible in \( C(10, 9)C(3, 1) = \frac{10!}{9!1!} \cdot \frac{3!}{2!1!} = 10 \cdot 3 = 30 \) ways; two women chosen — in \( C(10, 8)C(3, 2) = \frac{10!}{8!2!} \cdot \frac{3!}{2!1!} = 45 \cdot 3 = 135 \) ways; three women chosen — in \( C(10, 7)C(3, 3) = \frac{10!}{7!3!} \cdot \frac{3!}{3!0!} = \frac{10 \cdot 9 \cdot 8}{1 \cdot 2 \cdot 3} = 120 \) ways. Altogether there are \( 30 + 135 + 120 = 285 \) possible choices.