Problem 8.1. Factor the following integers into powers of primes: 30, 36, 81, 11, 111, 1111, 210, 625, 667, 223, 297, 10000.

Problem 8.2.
   a) Find the divisors of 30.
   b) Find the divisors of 36.
   c) Find the common divisors of 36 and 30.
   d) Compute $\gcd(36, 30)$.

Problem 8.3. Prove that $d | a$ implies that $d | ac$ for any $c$.

Problem 8.4. Prove that $\gcd(a, b) = \gcd(a - b, b) = \gcd(a + b, b) = \gcd(a + 2b, b)$.

Problem 8.5. Show that if $d | a$ and $d | b$ then $d | (a^2 + b^2)$.

Problem 8.6. If $\gcd(a, b) > 1$ and $\gcd(c, b) > 1$, is it true that $\gcd(a, c) > 1$?

Problem 8.7. Use the Euclidean algorithm to compute $\gcd(232, 64)$, $\gcd(342, 232)$, $\gcd(213, 263)$, and $\gcd(1714, 1814)$. Show each intermediate step in the algorithm.

Problem 8.8. Write down the multiplication and addition tables for $\mathbb{Z}_5$ and $\mathbb{Z}_6$. Do you notice a fundamental difference in the multiplication tables?

Problem 8.9. In $\mathbb{Z}_{100}$, find the multiplicative inverse of 97.

Problem 8.10. Suppose that $cx \equiv cy \mod n$ and that $c \not\equiv 0 \mod n$ ($c, x, y, n$ are integers). Prove or disprove: $x \equiv y \mod n$.

Problem 8.11. The least common multiple of two integers $a, b$, denoted $\text{lcm}(a, b)$, is the smallest positive integer that is a multiple of $a$ and $b$. Prove that for any two integers $a$ and $b$,

$$\gcd(a, b)\text{lcm}(a, b) = ab.$$