In order to receive full credit, you must show all the necessary work for each problem using correct notation. Your work must be clear and organized. And, all answers must be simplified unless stated otherwise. Also, remember that Valentine’s Day is in one month.

1. (2 Points) Complete the following derivative statements:

(a) \( \frac{d}{dx} \tan^{-1}(x) = \frac{1}{1 + x^2} \)

(b) \( \frac{d}{dx} \ln|x| = \frac{1}{x} \)

(c) \( \frac{d}{dx} \sec(x) = \sec(x) \tan(x) \)

(d) Suppose \( f(x) \) and \( g(x) \) are differentiable. \( \frac{d}{dx} (f(x)g(x)) = f(x)g'(x) + f'(x)g(x) \)

2. (3 Points) Suppose \( f(x) \) is a continuous non-negative function which satisfies the following:

<table>
<thead>
<tr>
<th>( x )</th>
<th>0</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
<th>6</th>
<th>7</th>
<th>8</th>
<th>9</th>
<th>10</th>
<th>11</th>
<th>12</th>
</tr>
</thead>
<tbody>
<tr>
<td>( f(x) )</td>
<td>3</td>
<td>9</td>
<td>13</td>
<td>25</td>
<td>12</td>
<td>4</td>
<td>20</td>
<td>18</td>
<td>11</td>
<td>7</td>
<td>3</td>
<td>100</td>
<td>5</td>
</tr>
</tbody>
</table>

Estimate the area under the graph of \( f(x) \) on the interval \([0, 12]\) using 3 rectangles of equal width and midpoints. \( \Delta x = \frac{12}{3} = 4 \)

Subintervals: \([0, 4]\) \([4, 8]\) \([8, 12]\)

\( x_{k+} = 2 \ 6 \ 10 \)

\( A \approx f(2)(4) + f(6)(4) + f(10)(4) \)
\( = (13)(4) + (20)(4) + (11)(4) \)
\( = (36)(4) \)
\( = \boxed{144} \)
3. (5 Points) Use sigma notation and the appropriate summation formulas to find the exact net signed area between the graph of \( f(x) = x+4 \) and the x-axis on the interval \([1, 3]\). Let \( x_k^* \) be the right endpoint of the \( k \)th subinterval (where all subintervals have equal width).

\[
\Delta x = \frac{3-1}{n} = \frac{2}{n} \quad \text{width of each rectangle}
\]

\[
x_k^* = a + k \Delta x = 1 + \frac{2k}{n} \quad \text{for } k = 1, 2, \ldots, n
\]

**Height of \( k \)th rect.:**

\[
f(x_k^*) = 5 + \frac{2k}{n}
\]

\[
A = \sum_{k=1}^{n} f(x_k^*) \Delta x = \sum_{k=1}^{n} \left(5 + \frac{2k}{n}\right) \left(\frac{2}{n}\right)
\]

\[
= \sum_{k=1}^{n} \left(\frac{10}{n} + \frac{4k}{n^2}\right)
\]

\[
= \frac{10}{n} \sum_{k=1}^{n} 1 + \frac{4}{n^2} \sum_{k=1}^{n} k
\]

\[
= \frac{10}{n} (n) + \frac{4}{n^2} \cdot \frac{n(n+1)}{2}
\]

\[
= 10 + \frac{2(n+1)}{n}
\]

\[
A = \lim_{n \to \infty} \sum_{k=1}^{n} f(x_k^*) \Delta x
\]

\[
= \lim_{n \to \infty} 10 + \frac{2(n+1)}{n}
\]

\[
= 10 + 2
\]

\[
\boxed{12}
\]
In order to receive full credit, you must show all the necessary work for each problem using correct notation. Your work must be clear and organized. And, all answers must be simplified unless stated otherwise. Also, remember that Valentine’s Day is in one month.

1. (2 Points) Complete the following derivative statements:

   (a) \( \frac{d}{dx} (\sin^{-1}(x)) = \frac{1}{\sqrt{1-x^2}} \)

   (b) \( \frac{d}{dx} (\ln |x|) = \frac{1}{x} \)

   (c) \( \frac{d}{dx} (\cot(x)) = -\csc^2(x) \)

   (d) Suppose \( f(x) \) and \( g(x) \neq 0 \) are differentiable. \( \frac{d}{dx} \left( \frac{f(x)}{g(x)} \right) = \frac{g(x)f'(x) - f(x)g'(x)}{g(x)^2} \)

2. (3 Points) Suppose \( f(x) \) is a continuous non-negative function which satisfies the following:

   \[
   \begin{array}{c|cccccccccccc}
   x & 0 & 1 & 2 & 3 & 4 & 5 & 6 & 7 & 8 & 9 & 10 & 11 & 12 \\
   f(x) & 3 & 9 & 13 & 25 & 12 & 4 & 20 & 18 & 11 & 17 & 3 & 100 & 5 \\
   \end{array}
   \]

   Estimate the area under the graph of \( f(x) \) on the interval \([0, 12]\) using 4 rectangles of equal width and left endpoints.

   \( \Delta x = 12 / 4 = 3 \)

   Subintervals: \([0, 3]\), \([3, 6]\), \([6, 9]\), \([9, 12]\)

   \[x_1, x_2, x_3, x_4 = 0, 3, 6, 9\]

   \[
   A \approx f(0)(3) + f(3)(3) + f(6)(3) + f(9)(3)
   = (3)(3) + (25)(3) + (20)(3) + (17)(3)
   = 195
   \]
3. (5 Points) Use sigma notation and the appropriate summation formulas to find the exact net signed area between the graph of \( f(x) = x+5 \) and the \( x \)-axis on the interval \([1, 4]\). Let \( x_k^* \) be the right endpoint of the \( k \)th subinterval (where all subintervals have equal width).

\[
\Delta x = \frac{4-1}{n} = \frac{3}{n}
\]

\[
\chi_k^* = a + k \Delta x = 1 + \frac{3k}{n} \quad \text{for } k=1, 2, \ldots, n
\]

Height of \( k \)th rectangle:

\[
f(x_k^*) = 6 + \frac{3k}{n}
\]

for \( k=1, 2, \ldots, n \)

\[
A = \sum_{k=1}^{n} f(x_k^*) \Delta x = \sum_{k=1}^{n} \left( 6 + \frac{3k}{n} \right) \left( \frac{3}{n} \right)
\]

\[
= \sum_{k=1}^{n} \frac{18}{n} + \frac{9}{n^2} \frac{k}{n}
\]

\[
= \frac{18}{n} \sum_{k=1}^{n} 1 + \frac{9}{n^2} \sum_{k=1}^{n} k
\]

\[
= \frac{18}{n} (n) + \frac{9}{n^2} \frac{n(n+1)}{2}
\]

\[
= 18 + \frac{9(n+1)}{2n}
\]

\[
A = \lim_{n \to \infty} \sum_{k=1}^{n} f(x_k^*) \Delta x
\]

\[
= \lim_{n \to \infty} 18 + \frac{9(n+1)}{2n}
\]

\[
= 18 + \frac{9}{2}
\]

\[
= \frac{27}{2}
\]