Quiz 4 Solutions

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1

Suppose that in one metropolitan area, 25% of all homeowners are insured against earthquake damage. Four homeowners are to be selected at random; let $X$ denote the number among the four who have earthquake insurance. Find the probability distribution of $X$.

If selecting a person with insurance is defined as a success, then since one person being insured is independent of another person being insured, we have a binomial distribution with $p = .25$ and $n = 4$. Then the probability distribution of $X$ is

$$P(X = k) = \binom{4}{k}(.25)^k(.75)^{4-k}$$

for $k = 0, 1, 2, 3, 4$ and 0 otherwise. Numerically, we have $P(0) = .3164, P(1) = .4212, P(2) = .2109, P(3) = .0469, P(4) = .003$.

2

Two fair six-sided dice are tossed independently. Let $M$ stand for the maximum of the two tosses. What is the pmf of $M$?

The easiest way to determine the pmf is to simply make a table with the first toss as rows and the second as columns. A pattern is apparent. A more straightforward approach is to denote the first toss by $X$ and the second by $Y$. Then
since the tosses are independent, we have
\[ P(M = k) = P(M \leq k \land M \neq k) = P(M \leq k) - P(M \leq k - 1) \]
\[ = P(X \leq k \land Y \leq k) - [P(X \leq k - 1 \land Y \leq k - 1)] \]
\[ = P(X \leq k) \cdot P(Y \leq k) - [P(X \leq k - 1) \cdot P(Y \leq k - 1)] \]
\[ = \frac{k}{6} \cdot \frac{k}{6} - \left[ \frac{k - 1}{6} \cdot \frac{k - 1}{6} \right] \]
\[ = \frac{k^2 - (k^2 - 2k + 1)}{36} \]
\[ = \frac{2k - 1}{36} \]
For \( k = 1, 2, 3, 4, 5, 6 \) and 0 elsewhere. Numerically, this is \( P(1) = 1/36, P(2) = 3/36, P(3) = 5/36, P(4) = 7/36, P(5) = 9/36, P(6) = 11/36. \)