I. A production facility employs 20 workers on the day shift, 15 workers on the swing shift, and 10 workers on the graveyard shift. A quality control consultant is to select 6 of these workers for in-depth interviews. The selection is made in such a way that any group of 6 workers has the same chance of being selected (drawing 6 without replacement from among 45).

A What is the probability that all 6 selected workers will be the day shift?

B What is the probability that all 6 selected workers will be the same shift?

C What is the probability that at least two different shifts will be represented among the selected workers.

D What is the probability that at least one of the shifts will be unrepresented in the sample of workers?

Solution

A The probability that all 6 selected workers will be the day shift is

\[ P_1 = \frac{\binom{20}{6}}{\binom{45}{6}}. \]

B The probability that all 6 selected workers will be the same shift is

\[ P_2 = \frac{\binom{20}{6}}{\binom{45}{6}} + \frac{\binom{10}{6}}{\binom{45}{6}} + \frac{\binom{15}{6}}{\binom{45}{6}}. \]

C The opposite event of the event that at least two different shifts will be represented among the selected workers is the event in B. Hence the probability is

\[ P_3 = 1 - P_2 = 1 - \frac{\binom{20}{6}}{\binom{45}{6}} - \frac{\binom{10}{6}}{\binom{45}{6}} - \frac{\binom{15}{6}}{\binom{45}{6}}. \]
Suppose that the event that only day shifts be unrepresented is $A_1$, only swing shifts be unrepresented is $A_2$, only graveyard shifts be unrepresented is $A_3$.

The probability that at least one of the shifts will be unrepresented in the sample of workers is

$$\Pr(A_1 \cup A_2 \cup A_3) = \Pr(A_1) + \Pr(A_2) + \Pr(A_3) - \Pr(A_1 \cap A_2) - \Pr(A_1 \cap A_3) - \Pr(A_2 \cap A_3) + \Pr(A_1 \cap A_2 \cap A_3)$$

where $\Pr(A_1) = \frac{10 + 15}{60}$, $\Pr(A_2) = \frac{20 + 10}{60}$, $\Pr(A_3) = \frac{20 + 15}{60}$, $\Pr(A_1 \cap A_2) = \frac{10}{60}$,

$\Pr(A_1 \cap A_3) = \frac{15}{60}$, $\Pr(A_2 \cap A_3) = \frac{20}{60}$, $\Pr(A_1 \cap A_2 \cap A_3) = 0$, $\Pr(A_1 \cap A_2 \cap A_3) = 0$, $\Pr(A_1 \cap A_2 \cap A_3) = 0$. 