You must show all of your work to receive full credit. You have fifty (50) minutes to complete this exam. (5 pts each)

1. a. Define Type I error.
   \[ \text{Rejecting } H_0 \text{ when } H_0 \text{ is true} \]

   b. Define Type II error.
   \[ \text{Not rejecting the } H_0 \text{ when } H_0 \text{ is false} \]

   c. Define \( \alpha \).
   \[ P(\text{type I error}) \text{ also called significance level of test} \]

   d. Define \( \beta \).
   \[ P(\text{type II error}) \]

2. Let \( \mu \) denote the true average tread life of a certain type of tire. Consider testing
   \[ H_0 : \mu = 30,000 \text{ vs. } H_a : \mu > 30,000 \]
   based on a sample of \( n = 16 \) from a normal population with \( \sigma = 1500 \). What is the probability of a type II error when \( \mu = 31,000 \) for a level .05 test? (15 pts)
   \[
P(\text{Z} \leq \text{Z}_{.05} + \frac{\mu_p - \mu}{\sigma/\sqrt{n}}) \]
   \[= P(\text{Z} \leq 1.645 - 1000) \]
   \[= P(\text{Z} \leq -1.021) = .1539 \]
3. The one hole test is used to test the manipulative skill of job applicants. This test requires subjects to grasp a pin, move it to a hole, insert it, and return for another pin. In one study, male college students were compared with experienced female industrial workers. Here are the data for the first minute of the test.

<table>
<thead>
<tr>
<th>Group</th>
<th>N</th>
<th>$\bar{X}$</th>
<th>S</th>
</tr>
</thead>
<tbody>
<tr>
<td>Students</td>
<td>750</td>
<td>35.12</td>
<td>4.31</td>
</tr>
<tr>
<td>Workers</td>
<td>412</td>
<td>37.32</td>
<td>3.83</td>
</tr>
</tbody>
</table>

It was expected that the experienced workers would outperform the students.

a. State the hypotheses for a statistical test of this expectation and perform the test.

Let $\mu_1$ = mean of students
$\mu_2$ = mean of workers

$H_0 : \mu_1 - \mu_2 = 0$
$H_a : \mu_1 - \mu_2 < 0$

Reject if $Z < -Z_{0.05}$

$Z = \frac{\bar{X}_1 - \bar{X}_2 - (\mu_1 - \mu_2)}{\sqrt{\frac{s_1^2}{m} + \frac{s_2^2}{n}}}$

$= \frac{-1.2}{\sqrt{0.0671 + 0.0622}} = \frac{-1.2}{0.3596} = -3.34$

Reject $H_0$.

b. Give a p-value and state your conclusions.

$p = \Phi(-3.34) = 0.0009$

As we can reject at $\alpha = 0.05$ b/c 0.0009 is the smallest $\alpha$ level at which we can reject $H_0$. 

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4. A university library ordinarily has a complete shelf inventory done once every year. Because of new shelving rules instituted the previous year, the head librarian believes it may be possible to save money by postponing the inventory. The librarian decides to select at random 1000 books from the library’s collection and have them searched in a preliminary manner. If evidence indicates strongly that the true proportion of misshelved or unlocatable books is less than .25, the inventory will be postponed.

a. What are appropriate null and alternative hypotheses? (5 pts)

\[ H_0: \ p = .25 \]
\[ H_A: \ p < .25 \]

b. Among the 1000 books searched, 190 were misshelved or unlocatable. Test the hypotheses and advise the librarian what to do, using \( \alpha = .05 \). (10 pts)

\[ \hat{p} = \frac{190}{1000} = .190 \]
\[ z = \frac{\hat{p} - p_0}{\sqrt{\frac{p_0(1-p_0)}{n}}} = \frac{.190 - .25}{\sqrt{\frac{.25(1-.25)}{1000}}} = -3.28 \]

Reject \( H_0 \) if \(-3.28 < -1.645 \), so reject \( H_0 \) and advise postponement.

so reject \( H_0 \) and advise postponement.

C. For the alternative value \( p = .2 \), compute \( \beta(.20) \). (10 pts)

\[ \beta(.20) = 1 - \Phi\left( \frac{.25 - .2 - 1.645(.0137)}{\sqrt{.2(.1 - .2)/1000}} \right) \]
\[ = 1 - \Phi\left( -2.18 \right) = 1 - .9854 = .0146 \]

D. For \( \hat{p} = \frac{x}{n} = .20 \), compute the P-value. (10 pts)

\[ z = \frac{.22 - .25}{\sqrt{.22(.78)/1000}} = \frac{-.03}{.015} = -2.29 \]
\[ \Phi(-2.29) = .0143 \]
5. How does energy intake compare to energy expenditure. 7 professional soccer players were measured and the following data resulted in MJ/day:

<table>
<thead>
<tr>
<th>Player</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
<th>6</th>
<th>7</th>
</tr>
</thead>
<tbody>
<tr>
<td>Expenditure</td>
<td>14.4</td>
<td>12.1</td>
<td>14.3</td>
<td>14.2</td>
<td>15.2</td>
<td>15.5</td>
<td>17.8</td>
</tr>
<tr>
<td>Intake</td>
<td>14.6</td>
<td>9.2</td>
<td>11.8</td>
<td>11.6</td>
<td>12.7</td>
<td>15.0</td>
<td>16.3</td>
</tr>
</tbody>
</table>

(hint recall \( s^2 = \frac{1}{n-1} \left[ \sum x_i^2 - \left( \frac{\sum x_i}{n} \right)^2 \right] \))

a. Test to see if there is a significant difference between intake and expenditure at a significance level of .01.

\[ \bar{d} = \bar{x}_d - \bar{x}_i = 1.76 \]

\[ s_{d \bar{d}} = \frac{s_d}{\sqrt{n}} = \frac{1.197}{\sqrt{7}} = 1.197/\sqrt{7} \]

\[ t = \frac{\bar{d} - \bar{d}_0}{s_{d \bar{d}}} = \frac{1.76}{1.197/\sqrt{7}} = 3.89 \]

Reject \( H_0 \) if \( t < -t_{\alpha/2, n-1} \) or \( t > t_{\alpha/2, n-1} \)

\[ 3.89 < -3.707 \quad 3.89 > 3.707 \]

Reject \( H_0 \) there is a difference!

b. Compute a 95% Confidence interval for the true difference between expenditure and intake of energy.

\[ \bar{d} \pm t_{.025, n-1} s_{d \bar{d}} (1.4524) \]

\[ = 1.76 \pm 2.447 (1.4524) \]

\[ = (1.653, 2.867) \]
6. The summary data on the ratio strength to cross-sectional area for knee extensors is
taken from the article “Knee Extensor and Knee Flexor Strength”

<table>
<thead>
<tr>
<th>Group</th>
<th>Sample Size</th>
<th>Sample Mean</th>
<th>Standard error</th>
</tr>
</thead>
<tbody>
<tr>
<td>Young men</td>
<td>13</td>
<td>7.47</td>
<td>.22</td>
</tr>
<tr>
<td>Elderly men</td>
<td>12</td>
<td>6.71</td>
<td>.28</td>
</tr>
</tbody>
</table>

Note: standard error = $s / \sqrt{n}$

a. Does this data suggest that the true average ratio for young men exceeds that for
elderly men? Carry out a test of hypotheses using a significance level of .05.

\[
T = \frac{\bar{X} - \bar{Y} - (\mu_1 - \mu_2)}{\sqrt{\frac{s_1^2}{m} + \frac{s_2^2}{n}}} \\
T = \frac{7.47 - 6.71}{\sqrt{.22^2 + .28^2}} = 2.13 \\
\]

\[v = (\frac{s_1^2}{m} + \frac{s_2^2}{n}) = (\frac{.22^2}{13} + \frac{.28^2}{12}) = 21.3 \Rightarrow \nu = 21\]

\[2.13 > 1.71 \text{ Reject } H_0\]

b. What assumptions must you make in order to perform this test?

Normality
Samples are independent