2 × 2 zero-one matrices

How many 2 × 2 zero-one matrices are there?

We have two options, 0 or 1, for each of the 4 entries. The total count is therefore

\[ 2 \times 2 \times 2 \times 2 = 2^4 = 16. \]

How many of these are invertible?

The condition of invertibility rules out proportional columns.

Since each entry is either 0 or 1, this excludes zero and identical columns.

So here is the full list:

\[
\begin{bmatrix}
    1 & 0 \\
    0 & 1
\end{bmatrix},
\begin{bmatrix}
    0 & 1 \\
    1 & 0
\end{bmatrix},
\begin{bmatrix}
    1 & 1 \\
    0 & 1
\end{bmatrix},
\begin{bmatrix}
    1 & 1 \\
    1 & 0
\end{bmatrix},
\begin{bmatrix}
    1 & 0 \\
    1 & 1
\end{bmatrix},
\begin{bmatrix}
    0 & 1 \\
    1 & 1
\end{bmatrix}.
\]

We see that 6 out of 16 matrices are invertible, the remaining 10 are not.

The chance that a 2 × 2 zero-one matrix happens to be invertible is thus \(3/8 < 1/2\).

A randomly selected 2 × 2 zero-one matrix is more likely to have no inverse.

In contrast, the chance that a 2 × 2 matrix with real entries is invertible is 1.

Two arbitrarily chosen vectors in the plane are unlikely to be parallel,

so a generic matrix \(\begin{bmatrix} a & b \\ c & d \end{bmatrix}\) does not have proportional columns.

Perhaps, this explains the terms singular (not invertible) and nonsingular (invertible).