1. Find the matrix of rotation by an angle of 60° in the counterclockwise direction.

2. Interpret the transformation $T(x) = \begin{bmatrix} 1 & 1 \\ -1 & 1 \end{bmatrix} x$ geometrically.

3. The matrix $\begin{bmatrix} -0.8 & -0.6 \\ 0.6 & -0.8 \end{bmatrix}$ represents a rotation. Find the angle of rotation in radians.

4. Find the matrix of the orthogonal projection onto the line in $\mathbb{R}^3$ parallel to $\begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}$.

5. Find the matrix of reflection about the line $x_1 = x_2$ in $\mathbb{R}^2$.

6. Let $A$ be a reflection matrix and $x$ a vector in $\mathbb{R}^2$. Set $v = x + Ax$ and $w = x - Ax$. Express $A(Ax)$ in terms of $x$. Express $Av$ in terms of $v$. Express $Aw$ in terms of $w$. What is the angle between $v$ and $w$? How does $v$ relate to the line of reflection?

7. Find the matrix of the orthogonal projection onto the line in $\mathbb{R}^3$ containing the unit vector $u = \begin{bmatrix} u_1 \\ u_2 \\ u_3 \end{bmatrix}$.

What is the sum of diagonal entries of the matrix?

8. Find the matrix of each of the following linear transformations of $\mathbb{R}^3$: (a) the orthogonal projection onto the $x_1, x_2$-plane (b) the reflection about the $x_1, x_3$-plane (c) the rotation about the $x_3$-axis through an angle of $\pi/2$, counterclockwise as viewed from the positive $x_3$-axis (d) the rotation about the $x_2$-axis through an angle of $\theta$, counterclockwise as viewed from the positive $x_2$-axis (e) the reflection about the plane $x_2 = x_3$.

9. Find the inverse of the matrix $\begin{bmatrix} 1 & 5 \\ 0 & 1 \end{bmatrix}$. Interpret your answer geometrically.

10. Find the matrix of each of the following linear transformations of $\mathbb{R}^2$: (a) scaling that transforms $\begin{bmatrix} 2 \\ -1 \end{bmatrix}$ into $\begin{bmatrix} 8 \\ -4 \end{bmatrix}$ (b) orthogonal projection that transforms $\begin{bmatrix} 2 \\ 3 \end{bmatrix}$ into $\begin{bmatrix} 2 \\ 0 \end{bmatrix}$ (c) the rotation that transforms $\begin{bmatrix} 0 \\ 5 \end{bmatrix}$ into $\begin{bmatrix} 3 \\ 4 \end{bmatrix}$ (d) the shear that transforms $\begin{bmatrix} 1 \\ 3 \end{bmatrix}$ into $\begin{bmatrix} 7 \\ 3 \end{bmatrix}$ (e) the reflection that transforms $\begin{bmatrix} 7 \\ -5 \end{bmatrix}$ into $\begin{bmatrix} -5 \\ 5 \end{bmatrix}$. 
1. Suppose a line $L$ through the origin (in $\mathbb{R}^2$) makes an angle of $\theta$ radians with the $x$-axis. Write the matrix of the reflection about $L$ in terms of $\theta$. Express your answer in a good form.

2. \[
\begin{bmatrix}
0.6 & 0.8 \\
0.8 & -0.6
\end{bmatrix}
\]
is a matrix of reflection about what line?

3. Find a nonzero $2 \times 2$ matrix $A$ such that $Ax$ is parallel to $\begin{bmatrix}1 \\ 2\end{bmatrix}$, for all $x = \begin{bmatrix}x_1 \\ x_2\end{bmatrix}$.

4. Find a nonzero $3 \times 3$ matrix $A$ such that $Ax$ is perpendicular to $\begin{bmatrix}1 \\ 2 \\ 3\end{bmatrix}$, for all $x = \begin{bmatrix}x_1 \\ x_2 \\ x_3\end{bmatrix}$.

5. One of the five matrices below represents an orthogonal projection onto a line and another represents a reflection about a line. Identify both and justify your choice.

\[
\begin{bmatrix}
1 & 2 & 2 \\
2 & 1 & 2 \\
2 & 2 & 1
\end{bmatrix}, \quad
\begin{bmatrix}
1 & 1 & 1 \\
1 & 1 & 1 \\
1 & 1 & 2
\end{bmatrix}, \quad
\begin{bmatrix}
2 & 1 & 1 \\
1 & 2 & 1 \\
1 & 1 & 2
\end{bmatrix}, \quad
\begin{bmatrix}
1 & 2 & 2 \\
2 & 1 & 2 \\
2 & 2 & 1
\end{bmatrix}, \quad
\begin{bmatrix}
-1 & 2 & 2 \\
2 & -1 & 2 \\
2 & 2 & -1
\end{bmatrix}
\]

6. Let $P$ and $Q$ be two perpendicular lines in the plane. Given $x = \begin{bmatrix}x_1 \\ x_2\end{bmatrix}$, what is the vector sum of orthogonal projections of $x$ onto $P$ and onto $Q$?

7. Let $P$ and $Q$ be two perpendicular lines in the plane. Given $x = \begin{bmatrix}x_1 \\ x_2\end{bmatrix}$, what is the relationship between the reflections of $x$ about $P$ and about $Q$?

8. Let $P$ and $Q$ be two lines through the origin in $\mathbb{R}^2$, forming an angle of $30^\circ$. Let $T(x)$ be obtained by reflecting $x$ about $P$ and then reflecting the result about $Q$. What is the angle between $x$ and $T(x)$? Find the matrix of $T$ and identify $T$ geometrically.