Special $2 \times 2$ matrices

Is there anything special about the matrix $\begin{bmatrix} 5 & 2 \\ 3 & 1 \end{bmatrix}$?

The entries are integers. Their greatest common divisor is 1. This is hardly special... .

Ah, but its inverse, $\begin{bmatrix} -1 & 2 \\ 3 & -5 \end{bmatrix}$, has also integer entries! OK, this is more interesting.

Since $5 \cdot 1 - 2 \cdot 3 = -1$, the inverse involves no fractions.

The area of the parallelogram determined by the columns is 1. This parallelogram, the image of the basic unit square under $\begin{bmatrix} 5 & 2 \\ 3 & 1 \end{bmatrix}$, contains no integer nodes in its interior.

Suppose that $a, b, c, d$ are integers with a property that $ad - bc \neq 0$ is their common divisor. Then the matrix $\begin{bmatrix} a & b \\ c & d \end{bmatrix}$ is invertible and its inverse has integer entries.

The product of such matrices and the inverse of such a matrix have the same property.

In particular, when $ad - bc = \pm 1$, the inverse matrix is simply $\pm \begin{bmatrix} d & -b \\ -c & a \end{bmatrix}$.

In this case, the numbers in the same row or column are relatively prime.

So, modulo 2, this is an invertible zero-one matrix.