**SUM OF INDEPENDENT STANDARD GAUSSIANS**

Suppose that $X$ and $Y$ are independent standard normal variables,

$$X, Y \sim \mathcal{N}(0, 1).$$

We will find the distribution of $X + Y$ using geometry.

By independence, the joint density function of $X$ and $Y$ is

$$f(x, y) = \frac{1}{\sqrt{2\pi}} e^{-x^2/2} \frac{1}{\sqrt{2\pi}} e^{-y^2/2} = \frac{1}{2\pi} e^{-(x^2+y^2)/2} = \frac{1}{2\pi} e^{-r^2/2}.$$

As a function of just the polar radius $r$, the joint density is rotationally symmetric.

For each $t$, the event $X + Y \leq t$ consists of all outcomes $(x, y)$ such that $x + y \leq t$. This is a half-plane bounded by the line $x + y = t$, which is at distance $|t|/\sqrt{2}$ from $(0, 0)$.

By the symmetry of joint density, the probability that $(X, Y)$ is in the half-plane $x + y \leq t$ is the same as the probability that $(X, Y)$ is in any other half-plane obtained by rotating $x + y \leq t$ about the origin.

In particular,

$$P(X + Y \leq t) = P(X \leq t/\sqrt{2}) = P(\sqrt{2} X \leq t).$$

Hence $X + Y$ and $\sqrt{2} X \sim \mathcal{N}(0, 2)$ have the same distribution. In other words, $X + Y$ is normal with mean $\mu = 0$ and variance $\sigma^2 = 2$. 