Chapter 6.4 Practice Problems

EXPECTED SKILLS:

- Be able to find the arc length of a smooth curve in the plane described as a function of x or as a function of y.

PRACTICE PROBLEMS:

For problems 1-3, compute the exact arc length of the curve over the given interval.

1. $y = 4x^{\frac{3}{2}} - 1$ from $x = \dfrac{1}{12}$ to $x = \dfrac{2}{9}$

   $\boxed{\dfrac{19}{54}}$

2. $y = \dfrac{x^2}{2} - \dfrac{\ln(x)}{4}$ for $2 \leq x \leq 4$

   $\boxed{6 + \dfrac{1}{4}\ln 2}$

3. $y = \dfrac{2}{3}(x^2 - 1)^{3/2}$ for $1 \leq x \leq 3$

   $\boxed{\dfrac{46}{3}}$

4. Consider the curve defined by $y = \sqrt{4 - x^2}$ for $0 \leq x \leq 2$.

   (a) Compute the arc length on the interval $[0, t]$ for $0 \leq t < 2$. (Your arc length will depend on t.)

   $\boxed{2\sin^{-1}\left(\dfrac{t}{2}\right)}$

   (b) Use your answer from part (a) to compute the arc length on the interval $[0, 2]$. (Hint: You will need to introduce a limit.)

   $\boxed{\pi}$

   (c) Confirm your answer from part (b) by using geometry.

   On the interval $[0, 2]$, the curve is $\dfrac{1}{4}$ of a circle with a radius of 2. So, the length should be $\dfrac{1}{4}$ of the circumference; that is, Length $= \dfrac{1}{4} \cdot 2\pi r \Big|_{r=2} = \dfrac{1}{4} \cdot 2\pi(2) = \pi$.

5. Consider $F(x) = \int_1^x \sqrt{t^2-1}\,dt$. Compute the arc length on $[1, 3]$

$\boxed{4}$

6. Consider the curve defined by $f(x) = \ln x$ on $[1, e^3]$

   (a) Set up but do not evaluate an integral which represents the length of the curve by integrating with respect to x.

   $$\boxed{L = \int_1^{e^3} \sqrt{1 + \frac{1}{x^2}}\,dx}$$

   (b) Set up but do not evaluate an integral which represents the length of the curve by integrating with respect to y.

   $$\boxed{L = \int_0^3 \sqrt{1 + e^{2y}}\,dy}$$

7. Consider the curve defined by $f(x) = \tan x$ on $\left[-\frac{\pi}{3}, \frac{\pi}{4}\right]$

   (a) Set up but do not evaluate an integral which represents the length of the curve by integrating with respect to x.

   $$\boxed{L = \int_{-\frac{\pi}{3}}^{\frac{\pi}{4}} \sqrt{1 + \sec^4 x}\,dx}$$

   (b) Set up but do not evaluate an integral which represents the length of the curve by integrating with respect to y.

   $$\boxed{L = \int_{-\sqrt{3}}^{1} \sqrt{1 + \frac{1}{(1+y^2)^2}}\,dy}$$

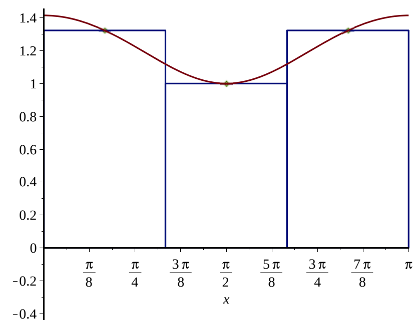
8. Consider the curve defined by $y = \sin x$ for $0 \le x \le \pi$.

   (a) Set up but do not evaluate an integral which represents the length of the curve.

   $$\boxed{\int_0^{\pi} \sqrt{1 + \cos^2 x}\,dx}$$

   (b) Estimate the value of your integral from part (a) by using a Midpoint Approximation with three rectangles of equal width.

Below is the graph of $y = \sqrt{1+\cos^2 x}$ on the interval $[0, \pi]$ along with three rectangles of equal width whose heights were determined by the function value at the midpoint of each resulting subinterval.



Using these rectangles, $\displaystyle\int_0^{\pi} \sqrt{1+\cos^2 x}\,dx \approx \frac{\pi}{3}\left(1+\sqrt{7}\right)$