Math 311: Exam 1

Name: Solutions

Directions:

- Do not open the exam until instructed to do so.
- You have 60 minutes to complete this exam. When time is called, stop writing immediately.
- You may not use any electronic devices except for pre-approved calculators. No cell phones or graphing calculators are allowed.
- Show all work. Answers without proper justification will not receive full credit, and partial credit will be awarded for significant progress towards the correct answer except for multiple choice questions. You must erase or cross off any work that you do not want graded. Do not simply give the answer to a problem as this may indicate cheating.
- Cheating of any form will result in a 0 for this exam. You may not look at another student's exam and you may not talk to another student during an exam.

Multiple Choice (no partial credit)

+8 1. An urn contains 10 balls: 4 red and 6 blue. A second urn contains 16 red balls and an unknown number of blue balls. A single ball is drawn from each urn. The probability that both balls are the same color is 0.44. Calculate the number of blue balls in the second urn.

(a) 4
(b) 20
(c) 24
(d) 44
(e) 64

+8 2. An insurance company pays hospital claims. The number of claims that include emergency room or operating room charges is 85% of the total number of claims. The number of claims that do not include emergency room charges is 25% of the total number of claims. The occurrence of emergency room charges is independent of the occurrence of operating room charges on hospital claims. Calculate the probability that a claim submitted to the insurance company includes operating room charges.

(a) 0.10
(b) 0.20
(c) 0.25
(d) 0.40
(e) 0.80

+8

3. An insurance company examines its pool of auto insurance customers and gathers the following information:

   - All customers insure at least one car.
   - 70% of the customers insure more than one car.
   - 20% of the customers insure a sports car.
   - Of those customers who insure more than one car, 15% insure a sports car.

   Calculate the probability that a randomly selected customer insures exactly one car and that car is not a sports car.

   (a) 0.13
   (b) 0.21
   (c) 0.24
   (d) 0.25
   (e) 0.30

Open Questions

4. Consider a 100 yard dash with 8 runners.

   (a) How many first, second, and third place finishers are possible?

   (b) How many top-three finishers are possible?

a) $\frac{8!}{(8-3)!} = 8 \cdot 7 \cdot 6 = 336$ 5pts

b) $\binom{8}{3} = \frac{8!}{5!\,3!} = \frac{8 \cdot 7 \cdot 6}{3 \cdot 2 \cdot 1} = 56$ 5pts

5. Suppose two cards are drawn without replacement from an ordinary deck of 52 cards. Find the probabilities of the following results:

   (a) Both cards are red.

   (b) A least 1 card is a face card (A Jack, Queen or King of any suit).

a) $\frac{\binom{26}{2}}{\binom{52}{2}} = .245$ 5pts

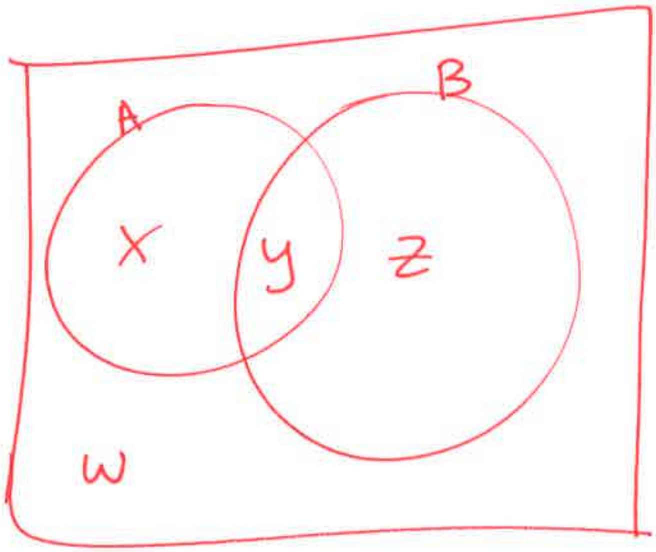
b) $\frac{\binom{12}{2} + \binom{12}{1}\binom{40}{1}}{\binom{52}{2}} = .412$ 5pts

6. A survey found that 69% of respondents like Pepsi or Coca-Cola. Let A represent the number of people who like Pepsi and B represent the number of people who like Coca-Cola. If $P(A \cup B^c) = .65$ and $P(B \cup (A^c \cap B^c)) = .76$, compute the following:

   (a) $P(A \cap B^c)$.

   (b) $P(A \cap B)$.

$$\begin{cases} x + y + w = .65 \\ w + y + z = .76 \\ x + y + z = .69 \\ \boxed{w = .31} \end{cases}$$ +2pts



$y + x = .34$

$y + z = .45$

$\boxed{y = .10}$ +2pts

$\boxed{x = .24}$ +2pts

$\boxed{z = .35}$ +2pts

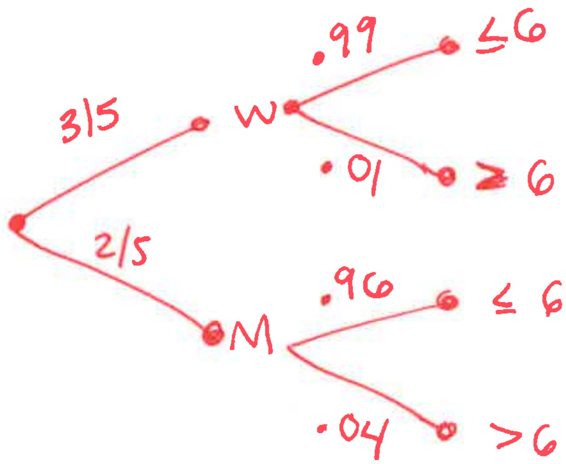
a) $\boxed{.24}$ +2pts 6pts

b) $\boxed{.10}$ +2pts 6pts

7. At a certain university, 4% of men are over 6 feet tall and 1% of women are over 6 feet tall. The total student population is divided in the ratio 3 women to every 2 men. Given this information, compute the following:

(a) What percentage of people at this university *under* 6 feet tall?

(b) If a student is selected at random from among all those *over* six feet tall, what is the probability that the student is a woman?



(Law of total probability) 2pts

a) $P(>6) = P(>6|M) \cdot P(M) + P(>6/W) \cdot P(W)$ 2pts

$= (.04)(.4) + (.01)(.6)$

$= .022$

$P(\leq 6) = 1 - P(>6) = \boxed{.978}$ 2pts

(Bayes theorem) 2pts

b) $P(W|>6) = \dfrac{P(>6|W)P(W)}{P(>6/W)P(W) + P(>6/M)P(M)}$ 2pts

$= \dfrac{(.6)(.01)}{(.6)(.01) + (.04)(.4)} = \dfrac{.006}{.022}$

$= \boxed{.273}$ 2pts

8. A student is taking an examination consisting of 3 questions selected randomly from a list of 100 questions. To pass, the student must answer all three questions correctly. What is the probability that this student will pass the examination if the student knows the answer to 90 of the 100 questions and does not know the answer to the remaining 10 questions? (Hint: apply the conditional probability formula twice).

$$P(Q_1 \cap Q_2 \cap Q_3) = P(Q_3 | Q_1 \cap Q_2) P(Q_1 \cap Q_2)$$
$$= P(Q_3 | Q_1 \cap Q_2) P(Q_2 | Q_1) P(Q_1)$$
$$= \frac{88}{98} \cdot \frac{89}{99} \cdot \frac{90}{100} = .73$$

OR: $\dfrac{\binom{90}{3}}{\binom{100}{3}}$

9. At a certain intersection, the light for eastbound traffic is red for 15 seconds, yellow for 5 seconds, and green for 30 seconds. Assuming that cars will drive through the intersection at any moment the light is green or yellow, find the probability that out of the next seven eastbound cars that arrive randomly at the light, more than two will be stopped by a red light.

Success = stopped want $P(S > 2) = 1 - P(S \leq 2)$

$p = .3$

$$P(S \leq 2) = \binom{7}{2}(.3)^2(.7)^5 + \binom{7}{1}(.3)^1(.7)^6$$
$$+ \binom{7}{0}(.3)^0(.7)^7$$

$$1 - P(S \leq 2) = \boxed{.353}$$

10. Suppose one of the numbers $2, \ldots, 12$ is chosen at random by throwing a pair of dice and adding the numbers shown. You win 9 dollars in case 2, 3, 11, or 12 is chose or lose 10 dollars if the outcome is 7. Otherwise, you do not win or lose anything. Let X be the random variable which denotes the amount won (or lost). Compute the following:

(a) The probability that you win money.

(b) The probability that you lose money.

a) $P(X>0) = P(X=9) = \frac{6}{36} = \boxed{\frac{1}{6}}$ 6pts

b) $P(X<0) = P(X=-10) = \frac{6}{36} = \boxed{\frac{1}{6}}$ 6pts