Math 102: Chapter 14.2
Optimization Word Problems

1. How would you divide a 10 inch string so that the product of the two lengths is maximized?

2. An open box is to be made from a 5-ft by 8-ft rectangular piece of sheet metal by cutting out squares of equal length from each of the four corners and bending up the sides. Find the maximum volume that the box can have.

3. A rectangular enclosure is to be constructed with an area of $600 \text{ ft}^2$. Three of the sides are to be made of wood with a cost of $7$ per foot. The fourth side is to be made of cement with a cost of $14$ per foot. What dimensions minimize the cost? What is the minimum cost?

4. A company manufactures and sells $x$ video phones per week. The weekly price demand is given by $p = 500 - 0.5x$ and the weekly cost function is given by $C(x) = 20,000 + 135x$.
   a) What price should the company charge to maximize the weekly revenue? What is the maximum weekly revenue?
   b) What is the maximum weekly profit? How much should the company charge and how many phones should be produced to realize the maximum weekly profit?

5. (HARDER PROBLEM!) A store can sell 20 bicycles per week at $400$ each. For each $10$ price reduction, they can sell 2 more bikes. The bikes cost the store $200$ each. Find the maximum profit. How many bikes must the store sell each week to achieve this maximal profit?
**Section 14.1: Absolute Extrema**

Recall: Two Types of Absolute Extrema Problems:

**Type 1: \( f(x) \) is continuous on a closed and bounded interval \([a, b]\).**
**Then \( f(x) \) has both an absolute maximum and an absolute minimum.**
**Use extreme value theorem (EVT).**

Extreme Value Theorem: A function \( f \) which is continuous on the closed and bounded interval \([a, b]\) is guaranteed to have both an absolute maximum and an absolute minimum. Moreover, these will either occur at the endpoints of the interval or at the critical points in \((a, b)\).

Strategy:
1. Evaluate \( f(a) \) and \( f(b) \).
2. Compute the critical points of \( f(x) \) in \((a, b)\) and evaluate the function at these points.
3. Now that we have the height of the function at the endpoints and at the critical points, we compare. The largest is the absolute maximum whereas the smallest is the absolute minimum.

**Type 2: \( f(x) \) is continuous on a some interval \( I \). (not \([a, b]\))**
**For example, \( I \) may be \((0, \infty)\) or \((1, 3)\).**

Strategy:
1. If \( f(x) \) has an absolute maximum or minimum, it must occur at a critical point. So, we first compute the critical points in the interval.
2. If \( f(x) \) has EXACTLY ONE critical point in the interval then
   i) If there is a local max at this critical point, there is also an absolute max at this critical point
   ii) If there is a local min at this critical point, there is also an absolute min at this critical point.

**Section 14.2: Optimization (Absolute Extrema Word Problems)**

Steps for the optimization word problems:
1. Read the problem carefully and make sure you understand what is given and what is unknown.
2. If possible, sketch a diagram and label the various parts.
3. Determine what is to be maximized or minimized and express it as a function of one variable.
4. Find the domain (including any physical restrictions) of the function.
5. If it is a closed & bounded interval, follow the directions for TYPE 1 from section 14.1. Otherwise, follow the directions for TYPE 2 from section 14.1.
6. Make sure you answer all of the questions that are asked. For example, make sure to give the dimensions which maximize the area or the maximal area, depending on what is asked.