Problem Set 5  
Due: Tuesday, October 23

**Problem 1.** Prove by induction on the number of faces that a plane graph $G$ is bipartite if and only if every face has even length.

**Problem 2.** Prove that every $n$-vertex plane graph isomorphic to its dual has $2n - 2$ edges. For all $n \geq 4$, construct a simple $n$-vertex plane graph isomorphic to its dual.

**Problem 3.** Prove that every simple planar graph with at least four vertices has at least four vertices with degree less than 6. Construct a simple planar graph $G$ with 8 vertices that has exactly four vertices with degree less than 6.

**Problem 4.** Prove that if $G$ is a color-critical graph, then the graph $G'$ generated from it by applying Mycielski’s construction is also color-critical (color-critical means $k$-critical for some $k$).

**Problem 5.** A *triangulation* is a simple plane graph where every face boundary is a 3-cycle. Prove that a triangulation is 3-colorable if and only if it is Eulerian. (Hint: Color the faces of $G$.)