Problem 1.1. List all of the subsets of $S = \{\{1, 2\}, 3, 4\}$.

Problem 1.2. How many subsets of the set $\{1, 2, \ldots, 2n\}$ are there which contain all the even numbers in this set? Justify your answer with a short proof.

Problem 1.3. Let $A$ be a set with $n$ elements and let $B$ be a set with $m$ elements.
   a) How many functions are there from $A$ to $B$?
   b) How many 1-1 functions from $A$ to $A$ are there?
   c) If $m < n$, how many 1-1 functions from $A$ to $B$ are there?
   d) If $m = 2$, how many onto functions from $A$ to $B$ are there?
Justify your answers with short proofs.

Problem 1.4.

Give an example of:
   a) A function on an infinite set that is 1-1, but not onto.
   b) A function on an infinite set that is onto, but not 1-1.

Problem 1.5. Prove the following distributive properties for set union and intersection:
   a) $A \cap (B \cup C) = (A \cap B) \cup (A \cap C)$
   b) $A \cup (B \cap C) = (A \cup B) \cap (A \cup C)$

Problem 1.6. Let $A, B,$ and $C$ be three sets and assume that $A$ is a subset of $C$. Prove that

$$A \cup (B \cap C) = (A \cup B) \cap C.$$ 

Show by example that the condition that $A$ is a subset of $C$ cannot be omitted.

Problem 1.7. Let $A$ be a set of size $n$ and $B$ be a set of size $n + 1$. How many pairs of functions are there $f : A \to B$ and $g : B \to A$ such that $g \circ f$ is the identity function on $A$? Justify your answer with a short proof.
Problem 1.8. Let $A$ be a set and let $\binom{A}{2}$ denote the set of all 2-element subsets of $A$. For example, if $A = \{1, 2, 4\}$, then $\binom{A}{2} = \{\{1, 2\}, \{1, 4\}, \{2, 4\}\}$. Which of the following statements is true? If false, give a counterexample, and if true, give a proof.

a) $\binom{A \cup B}{2} = \binom{A}{2} \cup \binom{B}{2}$

b) $\binom{A \cup B}{2} \supseteq \binom{A}{2} \cup \binom{B}{2}$

c) $\binom{A \cap B}{2} = \binom{A}{2} \cap \binom{B}{2}$

d) $\binom{A \cap B}{2} \subseteq \binom{A}{2} \cap \binom{B}{2}$