Problem 4.1.  

a) How many permutations of 7 are there so that the first three numbers are increasing (an example of such a permutation is 2571436)?

b) How many strings of length 5 are there in the alphabet 1, 2, . . . , 7 such that the first three numbers are increasing (an example of such a string is 25714)?

Problem 4.2.  
Compute $|\{A \subseteq \{1, 2, \ldots, 10\} : |A \cup \{1, 17\}| = 5\}|$. In other words, determine the number of subsets $A$ of $\{1, 2, \ldots, 10\}$ such that $A \cup \{1, 17\}$ has size 5.

Problem 4.3.  
A lattice path from a point $(a, b)$ to a point $(c, d)$ is a path from $(a, b)$ to $(c, d)$ using only steps of length 1 going east and steps of length 1 going north. The figure below depicts a lattice path from the point $(0, 0)$ to the point $(5, 6)$.

![Lattice Path Diagram]

a) How many lattice paths are there from $(0, 0)$ to $(5, 6)$?

b) How many lattice paths are there from $(0, 0)$ to $(5, 6)$ that go through the point $(3, 3)$?

Problem 4.4.  
We proved the following identity in class (for $n, m \geq 0$ and $n + m \geq k$):

$$
{n \choose 0} \binom{m}{k} + \binom{n}{1} \binom{m}{k-1} + \cdots + \binom{n}{k-1} \binom{m}{1} + \binom{n}{k} \binom{m}{0} = \binom{n + m}{k}.
$$

Give another proof using the binomial theorem and the fact that $(x + 1)^{n+m} = (x + 1)^n(x + 1)^m$. 

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Problem 4.5. Suppose we toss a fair coin three times in a row. What is the sample space $S$ that models this situation? Let $E$ be the event that exactly two of the tosses are heads. Write out $E$ explicitly as a subset of $S$. What is $P(E)$, i.e., the probability of getting exactly two heads?

Problem 4.6. Suppose you roll a blue six-sided die and a red six-sided die. Write down the event that a 6 appears on at least 1 of the dice (explicitly as a subset of the sample space). What is the probability of this event?

Problem 4.7. If you roll three six-sided dice, what is the probability that the result sums to 6?

Problem 4.8. Find the number of hands of the type straight flush: five cards of the same suit with consecutive values (we take the convention that A can be high or low but cannot wrap around, so $\{AS, 2S, 3S, 4S, 5S\}$ is a straight flush but $\{QC, KC, AC, 2C, 3C\}$ is not). Find the probability of being dealt such a hand.

Problem 4.9. a) List all the hands that consist of three jacks and two sevens (one such hand is $\{JS, JD, JH, 7C, 7S\}$).
   b) How many such hands are there? Show how to count this quickly using binomial coefficients.
   c) How many hands are there that consist of three jacks and a pair with value $A, 2, 3,$ or $4$ (one such hand is $\{JS, JD, JH, 2C, 2S\}$, another such hand is $\{JC, JD, JH, 4C, 4H\}$)?