Homework Set 6

Math 221 — Fall 19

Due Wednesday, November 13

Problem 6.1. Determine if the graphs below are isomorphic. If they are not isomorphic, show this by exhibiting some substructure that one has but not the other; if they are isomorphic, show this by labeling the vertices of both graphs by \( \{1, 2, \ldots, 8\} \) such that \( \{i, j\} \) is an edge in the graph on the left if and only if it is an edge in the graph on the right.

Problem 6.2. Use Kruskal’s Algorithm to find the minimum cost spanning tree in the following weighted graph. What is the first edge examined by the algorithm that is not selected to be part of the spanning tree, and why is it not selected? What is the second edge examined by the algorithm that is not selected to be part of the spanning tree, and why is it not selected?

Problem 6.3. Your friend is running Kruskal’s algorithm on the graph below and has so far selected the bold edges for the spanning tree. You can see all the edge weights except the weight
labeled \( w \). The next edge your friend selects is the one with weight \( w \). Assuming that the edge weights in the graph are distinct integers, what are the possibilities for \( w \)?

![Graph with edge labeled w](image)

**Problem 6.4.** The **chromatic number** \( \chi(G) \) of a graph \( G \) is the smallest \( k \) such that \( G \) is \( k \)-colorable.

a) Determine the chromatic number of the cube.

b) Determine the chromatic number of the dodecahedron.

c) Determine the chromatic number of the wheel on 8 vertices, shown below.

![Wheel on 8 vertices](image)

**Problem 6.5.** A **saturated hydrocarbon** is a molecule formed from \( k \) carbon and \( l \) hydrogen atoms by adding bonds between atoms such that each carbon is in 4 bonds, each hydrogen atom is in 1 bond, and no sequence of bonds forms a cycle of atoms. Prove that \( l = 2k + 2 \).

**Problem 6.6.** In the graph below, find a bipartite subgraph with the maximum number of edges. Explain why there is no bipartite subgraph with more edges.

![Graph with bipartite subgraph](image)

**Problem 6.7.** Prove or disprove: If \( u \) and \( v \) are the only vertices of odd degree in a graph \( G \), then \( G \) contains a path with ends \( u \) and \( v \).