Homework Set 6

MATH 221 — FALL 23

Due Wednesday, November 15

PROBLEM 6.1. Let $G$ be the graph below. Find examples of the following structures in $G$:

a) Find a path with 7 edges contained in $G$.

b) List all 4-cycles contained in $G$.

c) For which $n$ is there an $n$-cycle contained in $G$?

d) Give an example of a walk in $G$ from $v_3$ to $v_8$ with an odd number of edges.

PROBLEM 6.2.

a) Is there a graph on 6 vertices with degree sequence 5, 3, 3, 2, 2, 2?

b) Is there a graph on 6 vertices with degree sequence 4, 3, 3, 2, 2, 2?

PROBLEM 6.3.

a) Show that if $d_1, d_2, \ldots, d_n$ is the degree sequence of a tree, then $d_1 + d_2 + \cdots + d_n = 2(n - 1)$.

b) Find a tree whose degree sequence is 4, 3, 1, 1, 1, 1, 1.

c) Is there a tree whose degree sequence is 5, 4, 2, 2, 1, 1, 1, 1, 1, 1? Either exhibit such a tree or prove that no such tree exists.

PROBLEM 6.4. In a class with nine students, each student sends valentine cards to three others. Determine whether it is possible that each student receives cards from the same three students to whom they sent cards.
PROBLEM 6.5. Determine if the graphs below are isomorphic. If they are not isomorphic, show this by exhibiting some substructure that one has but not the other; if they are isomorphic, show this by labeling the vertices of both graphs by \( \{1, 2, \ldots, 8\} \) such that \( \{i, j\} \) is an edge in the graph on the left if and only if it is an edge in the graph on the right.

![Graphs for Problem 6.5](image)

PROBLEM 6.6. The **chromatic number** \( \chi(G) \) of a graph \( G \) is the smallest \( k \) such that \( G \) is \( k \)-colorable.

a) Determine the chromatic number of the cube (the graph on the left in problem 6.5).

b) Determine the chromatic number of the wheel on 8 vertices, shown below.

![Wheel Graph](image)

PROBLEM 6.7. A **saturated hydrocarbon** is a molecule formed from \( k \) carbon and \( l \) hydrogen atoms by adding bonds between atoms such that each carbon is in 4 bonds, each hydrogen atom is in 1 bond, and no sequence of bonds forms a cycle of atoms. Prove that \( l = 2k + 2 \).

PROBLEM 6.8. In the graph below, find a bipartite subgraph with the maximum number of edges. Explain why there is no bipartite subgraph with more edges.

![Bipartite Graph](image)

PROBLEM 6.9. Prove or disprove: If \( u \) and \( v \) are the only vertices of odd degree in a graph \( G \), then \( G \) contains a path with ends \( u \) and \( v \).