Homework Set 6

Math 222 — Fall 2021

Due Wednesday, November 17

Problem 6.1. Use Kruskal’s Algorithm to find the minimum cost spanning tree in the following weighted graph. What is the first edge examined by the algorithm that is not selected to be part of the spanning tree, and why is it not selected? What is the second edge examined by the algorithm that is not selected to be part of the spanning tree, and why is it not selected?

Problem 6.2. Your friend is running Kruskal’s algorithm on the graph below and has so far selected the bold edges for the spanning tree. You can see all the edge weights except the weight labeled $w$. The next edge your friend selects is the one with weight $w$. Assuming that the edge weights in the graph are distinct integers, what are the possibilities for $w$?

Problem 6.3. A saturated hydrocarbon is a molecule formed from $k$ carbon and $l$ hydrogen atoms by adding bonds between atoms such that each carbon is in 4 bonds, each hydrogen atom is in 1 bond, and no sequence of bonds forms a cycle of atoms. Prove that $l = 2k + 2$. 
Problem 6.4. Prove or disprove: If $u$ and $v$ are the only vertices of odd degree in a graph $G$, then $G$ contains a path with ends $u$ and $v$.

Problem 6.5. For a spanning tree $T$ in a weighted graph, let $m(T)$ denote the maximum among the weights of the edges in $T$. Let $x$ denote the minimum of $m(T)$ over all spanning trees of a weighted graph $G$. Prove that if $T$ is a spanning tree in $G$ with minimum total weight, then $m(T) = x$ (in other words, $T$ also minimizes the maximum weight). Construct an example to show that the converse is false. (Comment: A tree that minimizes the maximum weight is called a bottleneck or minimax spanning tree.)

Problem 6.6. Let $T_1, \ldots, T_k$ be subtrees of a tree $T$ such that for all $1 \leq i < j \leq k$ the trees $T_i$ and $T_j$ have a vertex in common. Show that $T$ has a vertex which is in all the $T_i$. 