Homework Set 8

MATH 221 — FALL 19

Due Wednesday, December 04

PROBLEM 8.1. Show that if \( d \mid a \) and \( d \mid b \) then \( d \mid (a^2 + b^2) \).

PROBLEM 8.2. If \( \gcd(a, b) > 1 \) and \( \gcd(c, b) > 1 \), is it true that \( \gcd(a, c) > 1? \)

PROBLEM 8.3. Use the Euclidean algorithm to compute \( \gcd(232, 64) \), \( \gcd(342, 232) \), \( \gcd(213, 263) \), and \( \gcd(1714, 1814) \). Show each intermediate step in the algorithm.

PROBLEM 8.4. Write down the multiplication and addition tables for \( \mathbb{Z}_5 \) and \( \mathbb{Z}_6 \). Do you notice a fundamental difference in the multiplication tables?

PROBLEM 8.5. In \( \mathbb{Z}_{100} \), find the multiplicative inverse of 97.

PROBLEM 8.6. Suppose that \( cx \equiv cy \mod n \) and that \( c \not\equiv 0 \mod n \) \((c, x, y, n \text{ are integers})\).
Prove or disprove: \( x \equiv y \mod n \).

PROBLEM 8.7. The least common multiple of two integers \( a, b \), denoted \( \text{lcm}(a, b) \), is the smallest positive integer that is a multiple of \( a \) and \( b \). Prove that for any two positive integers \( a \) and \( b \),

\[
\gcd(a, b)\text{lcm}(a, b) = ab.
\]

PROBLEM 8.8. a) Compute \( 10\%7, 100\%7, 1000\%7, 10000\%7, 100000\%7, 1000000\%7 \).

b) Use part a) to quickly determine \( 9001073\%7 \).

PROBLEM 8.9. Use the extended Euclidean algorithm to compute \( \gcd(342, 232) \); show each intermediate step in the algorithm and use this to find integers \( x \) and \( y \) such that \( 342x + 232y = \gcd(342, 232) \). Do the same for \( \gcd(1714, 1814) \).

PROBLEM 8.10. An element \( k \) of \( \mathbb{Z}_n \) is invertible if there is an \( a \in \mathbb{Z}_n \) such that \( ka \equiv 1 \mod n \). Find the inverses of all the invertible elements of \( \mathbb{Z}_{26} \).

PROBLEM 8.11. Do there exist integers \( x \) and \( y \) such that \( 4x + 6y = 3 \)? Either find a solution or say why none exists.