Homework Set 9

MATH 201 — FALL 2022

Due November 30 (Section 7)/ December 01 (Section 2)

Turn in the following book problems as well as problems 9.1–9.4 below.

Book problems from 6th edition:
Section 5.2 Problems 7,9,18.
Section 5.3 Problems 5, 9, 13, 16, 32, 33, 34.
Section 5.5 Problems 3, 8, 15, 27.

Or if you are using the 5th edition, these same problems are:
Section 5.2 Problems 7,9,18.
Section 5.3 Problems 5, 9, 13, 16, 26, 27, 28.
Section 5.5 Problems 3, 8, 15, 23.

Problem 9.1. Diagonalize the matrix $A = \begin{bmatrix} 4 & 0 & 0 & 0 \\ 0 & 4 & 0 & 0 \\ 0 & 0 & 2 & 0 \\ 1 & 0 & 0 & 2 \end{bmatrix}$.

Problem 9.2. Let $A = \begin{bmatrix} 1 & 4 & 5 \\ 0 & 0 & -2 \\ 0 & 0 & -1 \end{bmatrix}$. Compute $A^{2015}$.

Problem 9.3. Let $A$ be a $3 \times 3$ matrix with characteristic polynomial $\lambda^2(1 - \lambda)$.

1. Prove that $0 < \text{rank}(A)$ and $\text{rank}(A) < 3$.

2. Give an example of a matrix $A$ with the given characteristic polynomial and $\text{rank}(A) = 1$.

3. Give an example of a matrix $A$ with the given characteristic polynomial and $\text{rank}(A) = 2$.

Problem 9.4. Suppose that $A = PBP^{-1}$ where $A$ and $B$ are $n \times n$ matrices and $P$ is an invertible $n \times n$ matrix. Suppose that $Bv = \lambda v$ for some nonzero $v \in \mathbb{R}^n$ and $\lambda \in \mathbb{R}$. Show that $\lambda$ is an eigenvalue of $A$ and find an eigenvector of $A$ corresponding to eigenvalue $\lambda$. 