Quiz 6
Wednesday, March 2

Name:

Directions: For this quiz, no justification is required and no partial credit will be given. Circle your final answers clearly.

Problem 1. Determine whether the following statements are true or false. Circle your answers clearly. Each of the three parts below is worth one point.

a) True/False: The graph $K_{10}$ (the complete graph on 10 vertices) has 45 edges.

$True$. $K_{10}$ has $\binom{10}{2} = 45$ edges.

b) True/False: The graph below is 3-colorable.

$False$. This graph is the wheel on 10 vertices which has chromatic number 4 by the same argument used for the wheel on 8 vertices (see homework solutions).

c) True/False: For all $n \geq 1$, the complete bipartite graph $K_{2,n}$ is planar.

$True$: 

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Problem 2. Short answer. No justification is required and no partial credit will be given. Circle your final answers clearly. Each of the three parts below is worth one point.

a) We are running Kruskal’s algorithm on the graph below. The edges in bold have already been selected. Circle the next edge selected by Kruskal’s algorithm.

Answer:

b) Suppose that a connected planar graph $G$ has 9 vertices and 12 edges. How many faces does $G$ have?

Answer: By Euler’s formula, $v - e + f = 2$, thus $9 - 12 + f = 2$, hence $f = 5$.

c) A connected planar graph $H$ has 8 vertices and all its vertices have degree 4. How many faces must there be in any planar drawing of $H$?

Answer: Since there are 8 vertices all of degree 4, by the degree sum formula,

$$
\sum_{v \in V(H)} d(v) = \sum_{v \in V(H)} 4 = 8 \cdot 4 = 32 = 2|E(H)|,
$$

thus $|E(H)| = 16$. By Euler’s formula, $v - e + f = 2$, thus $8 - 16 + f = 2$, hence $f = 10$. 