

MATH 723

Spring 2007

Mathematical Neuroscience

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Lectures: R 6-9, Randel Hall 120
Office hours: R 5-5:50 and by appointment
Course webpage: <http://www.math.drexel.edu/~medvedev/classes/2006/math723/>
for the webpage from the last year, see
<http://www.math.drexel.edu/~medvedev/classes/2005/math680/>

Course description Computational neuroscience is a rapidly growing field of science with promising applications to physiology, medicine, and psychology, to name a few. It uses mathematical and computational modeling for studying how the nervous system functions. After a classical series of papers by Hodgkin and Huxley, nonlinear differential equations became a common framework for modeling electrical activity in neural cells. Now the language and methods of the theory of differential equations and applied dynamical systems are indispensable parts of the theoretical neuroscience.

The course focuses on mathematical concepts and techniques used in computational neuroscience. It is designed to provide students with necessary mathematical background for formulating, simulating, and analyzing models of individual neurons and neural networks. The course will serve as an introduction to the theory of nonlinear differential equations and applied dynamical systems in the context of neuronal modeling. The topics to be covered include a review of basic facts about the electrophysiology of neural cells, analysis of the conductance based models, neural excitability, bursting, models for synaptically coupled cells, and compartmental models, as well as a number of mathematical techniques such as phase plane analysis, slow-fast decomposition, and elements of the bifurcation theory. The students will learn basic models of excitable membranes such as Hodgkin-Huxley, Morris-Lecar, and FitzHugh-Nagumo models. They will acquire hands-on experience in numerical simulations of differential equations using XPPAUT. The course is aimed at graduate and advanced undergraduate students interested in computational biology and applied mathematics.

Content

Week I Electrophysiology of neural cells. Hodgkin-Huxley (HH) and Morris-Lecar models: derivation; action potential, rhythmic firing; numerical simulations using XPPAUT.

Week II 2D reduction of the HH system of equations. FitzHugh-Nagumo model. Slow-fast decomposition.

Week III Elements of the theory for differential equations: planar linear systems, examples of 2D nonlinear systems, linearization, phase-plane analysis.

Week IV-V Elements of the bifurcation theory: saddle-node, Andronov-Hopf, and homoclinic bifurcations. Applications of the bifurcation theory to classification and analysis of neuronal excitability.

Week VI Bursting: models of bursting neurons, classification, analysis.

Week VII-VIII Spatially extended HH model. Propagation phenomena: traveling fronts and pulses. Compartmental models.

Week IX-X Neuronal networks: synaptic connections, fast-threshold modulation, half-center oscillator.

Final Week Student presentations.

Prerequisites: knowledge of Calculus, Linear algebra, Differential Equations at an undergraduate level

Text: E.M. Izhikevich, *Dynamical Systems in Neuroscience: The Geometry of Excitability and Bursting*, The MIT Press, 2007.

Technology: I will use XPPAUT, a package for numerical analysis differential equations. This is free software and is available to download from www.pitt.edu/~phase/. Most homework assignments will contain problems to be solved numerically using XPPAUT. I will explain how to use XPPAUT in class. The program is installed on pi.math.drexel.edu. To run XPPAUT on *pi*, use command `xppaut`. All students registered for this course have accounts on this server. If you have problem logging in to *pi* please contact Gene Phan: cgp23@drexel.edu). To run XPPAUT under Windows, you will also need to install an X-server. X-Win32, an X-server provided by Drexel University, is available from <https://software.drexel.edu>.

Homework: Typically, I will assign a set of homework problems after every lecture and will collect them in class approximately every other week. Students are encouraged to discuss the homework and to work together on the problems. However, each student is responsible for final preparation of her/his homework papers. The presentation is important. Solutions should be presented in the order the problems were assigned. Every solution should be given a concise but sufficient explanation and written up legibly.

Assessment: Your final grade will be based on your performance on homework projects and on the final project.

A few useful references:

Physiology:

B Hill, Ion Channels of Excitable Membranes, third edition.

D.A. McCormick, Membrane properties and neurotransmitter actions, in G.M. Shepherd, ed.: *The Synaptic organization of the Brain*, fourth edition, Oxford University Press, 1998.

Computational Neuroscience:

P Dayan and LF Abbott, Theoretical Neuroscience.

J Keener and J Sneyd, Mathematical Physiology

CP Fall, ES Marland, JM Wagner, and JJ Tyson, Computational Cell Biology

XPP:

B. Ermentrout, *Simulating, Analyzing, and Animating Dynamical Systems: a Guide to XPPAUT for Researches and Students*, SIAM, Philadelphia, 2002.

Also see Bard Ermentrout's home page, <http://www.pitt.edu/~phase/>, for supplementary materials on XPPAUT (xpp codes, a tutorial; pictures and movies generated using XPPAUT).