

Mathematical Neuroscience

Homework Project 1. Due TBA in class.

The material for this part of the course is covered in §§ 2.1, 2.2, 2.4, and 2.5 in the course notes by P. Eckhoff and P. Holmes and in the handouts ‘Elements of the theory of differential equations’ and ‘Elements of the bifurcation theory’.

Problems:

1. Follow the lines of the example analyzed in Sec. 3.2 and 3.3 in the handout (Elements of the theory of differential equations) to analyze the system of equations of motion of a pendulum:

$$\begin{cases} \dot{x} = y, \\ \dot{y} = -\sin x. \end{cases} \quad (1)$$

In particular,

- a. Use the graph of the potential energy $U(x) = 1 - \cos x$ to construct the phase plane of (1).
 - b. Locate the fixed point. Linearize (1) around the fixed points. Sketch the phase portraits for the linearized systems and compare them with the phase portrait of (1). State if the Hartman-Grobman theorem applies to the linearized systems. Discuss stability of the fixed points.
2. **Problem 2** is posted on the web (see Lecture 2, below Topics). Follow the steps described in this problem. After that, modify the parameters of the model and repeat all the steps. In your report, include your codes and the representative plots.
 3. Write a matlab code for plotting the phase portrait for the equation of pendulum in Problem 1. Compare it with your solution of Problem 1. Include the code and the phase plane plot.