MATH 723 Spring 2008

Mathematical Neuroscience

Homework Project 3. May 12 in class.

Read:

- Course notes for Lectures 3,4, and 5 (see the course website)
- Chapter 2 and §4.3 in the textbook.

Problems:

1. `Saddle-node on an invariant circle bifurcation.` Consider a system of equations written in polar coordinates:

$$\dot{\rho} = \rho(1-\rho^2), \tag{1}$$
$$\dot{\theta} = 1+\mu-\sin\theta, \ \ \rho \geq 0, \ \theta \in S^1. \tag{2}$$

   Sketch (by hand) the phase portraits for positive and negative values of μ near 0 and for $\mu = 0$. for small $\mu > 0$, estimate the period of the limit cycle as a function of μ.

2. `The homework project for Lecture 4 'The Hodgkin-Huxley model'` (see notes for this lecture). The matlab code for the numerical experiments with the HH system of the previous subsection is given in the Appendix to this lecture. Modify this code to study the voltage responses in the ML system. For the ML system, use $(V_0, n_0) = (-60.855, 0.01495)$ as the initial condition. Find the values for the amplitude and the duration of stimulation which yield sub- and superthreshold responses, as well as trains of AP. Repeat this numerical experiment for $\phi = 0.02$. Describe the effect of changing ϕ on the trains of AP generated for prolonged stimulation. To make sure, that you entered the parameters correctly, compare your numerics with that in Figure 4 for the same values of parameters.

3. `The homework project for Lecture 5 ''The Hodgkin-Huxley model'` (see notes for this lecture).

   a. Plot the phase portrait for the ML model. Determine whether it is a type I or type II model. Change the value(s) of some parameter(s) in this model to make it of a different type.

   b. Use phase plane analysis to discuss two mechanisms for the action potential generation (excitability) in the Morris-Lecar model.