

MATH 723

Spring 2008

Mathematical Neuroscience

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Lectures: M 6-9, Matheson 310
Office hours: M 4:30-5:50 or by appointment
Course webpage: <http://www.math.drexel.edu/~medvedev/classes/2007/math723/>
for the webpage from the last year, see
<http://www.math.drexel.edu/~medvedev/classes/2006/math723/>

Course description Computational neuroscience is a rapidly growing field of science with promising applications to physiology, medicine, and psychology, to name a few. It uses mathematical modeling for studying how the nervous system functions. After a classical series of papers by Hodgkin and Huxley, nonlinear differential equations became a common framework for modeling electrical activity in neural cells. Now the language and methods of the dynamical systems theory are indispensable parts of the theoretical neuroscience. The course focuses on mathematical concepts and techniques used in computational neuroscience. It is designed to provide students with necessary mathematical background for formulating, simulating, and analyzing models of individual neurons and small neural networks. The course will also serve as an practical introduction to basic methods for studying nonlinear differential equations in the context of neuronal modeling. The topics to be covered include a review of basic facts about the electrophysiology of neural cells, analysis of the firing rate and conductance based models, neuronal excitability, bursting, models for small networks of coupled neurons, and (if time permits) compartmental models, as well as a number of mathematical techniques such as phase plane analysis, slow-fast decomposition, and elements of the bifurcation theory. The students will learn basic models of excitable membranes such as Hodgkin-Huxley, Morris-Lecar, and FitzHugh-Nagumo models. They will acquire hands-on experience in numerical simulations of differential equations using Matlab (no prior experience in MATLAB programming is expected). The course is aimed at graduate and advanced undergraduate students interested in computational biology and applied mathematics.

Prerequisites: knowledge of Calculus, Linear algebra, Differential Equations at an undergraduate level.

Sources: I will use the combination of the material from the Chapters 2-3 of the lecture notes by P. Eckhoff and P. Holmes 'Mathematical Neuroscience' (Course notes, Fall 2006, Princeton University) and the text by E.M. Izhikevich, *Dynamical Systems in Neuroscience: The Geometry of Excitability and Bursting*, The MIT Press, 2007. The former will be distributed in class. This material will be supplemented by the handouts covering selected topics and examples and research articles, which will be distributed in class and posted on the course website.

Homework: There will 3 – 4 homework projects assigned in the course of this class. They will combine mathematical problems and modeling assignments to be done in Matlab. Students are encouraged to discuss the homework and to work together on the problems. However, each student is responsible for the final preparation of her/his homework papers. The presentation is important. Solutions should be presented in the order the problems were assigned. Every solution should be given a concise but sufficient explanation and written up legibly. Solutions of the problems involving Matlab programming should be supplemented with the corresponding Matlab codes.

Final project: The final project will be based on reading a research paper relevant to the material covered in this course and presenting it in class. The presentation should cover the motivation and the main results of a given study. Substantial portions of the mathematical analysis and/or computational modeling that are reported in the paper should be reproduced. The final project may be prepared individually or in groups of two students depending on the size of the article. The students are welcome to choose papers for presentation on their own provided they are approved by the instructor.

Technology: We will gradually develop skills for using Matlab through examining examples in class and solving homework problems. Many homework assignments will contain problems to be solved numerically in Matlab.

Assessment: The final grade will be based on the performance on homework projects and on the final project.

Tentative plan

Weeks I-III Mathematical techniques: linear systems of differential equations, linearization of nonlinear systems, equilibria and limit cycles, Poincare map, planar systems, phase plane analysis, elements of the bifurcation theory.

Week IV-VI Conductance based models: Electrophysiology of neural cells. Hodgkin-Huxley and Morris-Lecar models: derivation; action potential, rhythmic firing. $2D$ reduction of the HH system of equations. FitzHugh-Nagumo model. Slow-fast decomposition. Neuronal excitability.

Week VII Bursting: models of bursting neurons, classification; analysis: reduction to one-dimensional maps, transitions between different dynamical modes in models of bursting neurons, chaos.

Week VIII-IX Small networks: synaptic connections, fast-threshold modulation, half-center oscillator, reduction to phase models. Time permitting, we will cover spatially extended HH model and related propagation phenomena.

Week X Student presentations.

A few useful references:

Physiology:

B Hill, Ion Channels of Excitable Membranes, third edition.

D.A. McCormick, Membrane properties and neurotransmitter actions, in G.M. Shepherd, ed.: *The Synaptic organization of the Brain*, fourth edition, Oxford University Press, 1998.

Computational Neuroscience:

P Dayan and LF Abbott, Theoretical Neuroscience.

CP Fall, ES Marland, JM Wagner, and JJ Tyson, Computational Cell Biology

C. Koch and I. Segev, eds, Methods in Neuronal Modeling

B. Ermentrout, *Simulating, Analyzing, and Animating Dynamical Systems: a Guide to XP-PAUT for Researches and Students*, SIAM, Philadelphia, 2002.

Nonlinear Dynamics:

S. Strogatz, *Nonlinear Dynamics and Chaos*, Westview Press, 2000.

Yu. Kuznetsov, *Elements of applied bifurcation theory*

Matlab :

DJ Higham and NJ Higham, Matlab Guide

P Pratap, Getting Started with Matlab.

Modern History of Neurosciece:

Eric R. Kandel, *In Search of Memory: The Emergence of a New Science of Mind*